One-dimensional transport equation models for sound energy propagation in long spaces: Simulations and experiments

Yun Jing and Ning Xiang

Graduate Program in Architectural Acoustics, School of Architecture, Rensselaer Polytechnic Institute, Troy, New York 12180

(Received 8 September 2009; revised 5 January 2010; accepted 7 January 2010)

In this paper, the accuracy and efficiency of the previously discussed one-dimensional transport equation models [Y. Jing et al., J. Acoust. Soc. Am. 127, 2312–2322 (2010)] are examined both numerically and experimentally. The finite element method is employed to solve the equations. Artificial diffusion is applied in the numerical implementation to suppress oscillations of the solution. The transport equation models are then compared with the ray-tracing based method for different scenarios. In general, they are in good agreement, and the transport equation models are substantially less time consuming. In addition, the two-group model is found to yield more accurate results than the one-group model for the tested cases. Lastly, acoustic experimental results obtained from a 1:10 long room scale-model are used to verify the transport equation models. The results suggest that the transport equation models are able to accurately model the sound field in a long space. © 2010 Acoustical Society of America. [DOI: 10.1121/1.3303981]

PACS number(s): 43.55.Br, 43.55.Ka [EJS] Pages: 2323–2331

I. INTRODUCTION

This paper verifies the one-dimensional transport equation models in long spaces using both numerical simulations and acoustical experimental results. A companion paper has presented theoretical formulations of a subset of one-dimensional transport equation models for acoustic prediction in long spaces. These models drastically simplify the three-dimensional exact model by reducing five variables to two (three in space, two in angle); thus, they are expected to be less time consuming in comparison with ray-tracing based methods or radiosity based methods when implemented numerically. On the other hand, a simple example of a sound field in free space has demonstrated that these models are fairly good approximations to the exact model. Particularly, since the diffusion equation can be derived as the asymptotic approximation of the transport equation in certain cases, the transport equation method will perform significantly better than the diffusion equation model recently applied in room-acoustics. For example, the transport equation takes the direct sound field into account while the diffusion equation fails to do so; therefore, the diffusion equation is only valid in the late time. The diffusion equation is inherently unsuitable for problems where the absorption involved is high, while the transport equation is much less restricted to this condition. The diffusion equation model assumes diffusely reflecting surface in the enclosure under investigation while the transport equation can handle explicitly partial specular and partial scattering reflections. In addition, solving the present one-dimensional transport equation does not necessarily take much longer time than solving the three-dimensional diffusion equation. This follow-up paper focuses on solving both the one- and two-group transport equation models numerically for different long spaces and compares the results with those obtained with a ray-tracing based method as well as experimental results obtained from a long room scale-model.

This paper is structured as follows: Sec. II briefly revisits the one-dimensional transport equation models for room-acoustic prediction in long spaces and then introduces solution methods used in solving the equations. Section III discusses simulation results obtained using the transport equation models in comparison with the ray-tracing based method. Section IV describes the scale-model experiments and compares the acoustical measurement results with those obtained using the two-group transport equation model. Section V concludes the paper.

II. ONE-DIMENSIONAL TRANSPORT EQUATION MODELS

A. Governing equations

This section briefly reviews the one-dimensional transport equation models for sound propagation in a long enclosure.

Based on the concept of geometrical acoustics, the sound angular flux \( \psi(r, \Omega, t) \) everywhere in a long space [with \( r=(x,y,z) \) and \( x \) being the longest dimension ranging from 0 to \( L \)] is shown to be the solution of a three-dimensional transport equation

\[
\frac{1}{c} \frac{\partial \psi(r, \Omega, t)}{\partial t} + \nabla \cdot \psi(r, \Omega, t) + \frac{M}{4\pi} \psi(r, \Omega, t) = \frac{Q(r, t)}{4\pi}, \quad 0 < x < L \quad (y,z) \in A,
\]

with appropriate boundary conditions, where \( \Omega \) is the unit vector in direction of flight, \( c \) is the speed of sound, \( M \) is the...
atmospheric attenuation constant, $Q$ is a source term, $t$ is the time, and $A$ is the cross section of the long space.

This radiative transport equation has the same form as the one used in the geometrical optic model. Both models assume that waves behave as particles: phonon and photon. The direct solution of this transport equation is the angular flux (sometimes called radiance), which can immediately lead to the sound intensity and sound pressure by applying an integral over all the angles. Mathematically, a similar radiative transport equation can be derived from the wave equation in the high frequency regime by using the Wigner distribution. The solution of this transport equation is the angular distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.

The direct solution of this transport equation is the angular distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.\(^\text{13}–\text{15}\) The solution of this transport equation is an equation in the high frequency regime by using the Wigner distribution.

Using the method of weighted residuals (Galerkin), the angular flux $\psi$ can be approximated by\(^\text{16}\)

$$\psi(x,y,z,\mu, \gamma, t) \approx \sum_{j=1}^{N} \alpha_j(y,z,\gamma) \psi_j(x,\mu, t), \quad (2)$$

where $\alpha_j$ are prescribed basis functions, $\psi_j$ are unknown expansion functions, $\mu$ and $\gamma$ are angular variables, and the three-dimensional exact transport equation can be reduced to a coupled set of one-dimensional transport equations. Keeping the first or the first two one-dimensional transport equations leads to the so-called one- and two-group models, respectively. For an omnidirectional point source at $(x_0, y_0, z_0)$, the one-dimensional transport equation model is written as

$$\frac{1}{c} \frac{d}{dt} \psi_i + \mu \frac{\partial \psi_i}{\partial x} + M \psi_i + \gamma i - \mu^2 \sum_{j=1}^{N} a_{ij} \psi_j \int_{-1}^{1} \sqrt{1 - \mu'^2} \psi_j(\mu') d\mu' + Q_i, \quad (3)$$

where $N$ (or $i$) $= 1, 2$ for one- and two-groups models, respectively, $R$ is the average energy reflection coefficient, $s$ is the average scattering coefficient,\(^\text{17}\) which expresses the energy fraction between nonspecular and specular reflections, and $a_{ij}$ and $b_{ij}$ are predefined functions associated with the basis functions

$$a_{11} = \left[ 1 - R(1-s) \right] \frac{L'}{\pi A'},$$

$$a_{12} = \left[ 1 - R(1-s) \right] \left[ u - u_0 \frac{L'}{\pi A'} \right],$$

$$a_{21} = \left[ R(1-s) - 1 \right] \frac{u_0 L'}{\pi A'},$$

$$a_{22} = \frac{u^2 v^2 L'}{\pi A'} + R(1-s) uv \left[ u - u_0 \frac{L'}{\pi A'} \right], \quad (4)$$

$$b_{ij} = \left[ \frac{L'/(\pi A')} {u - u_0 L'/(\pi A')} - u_0 [u - u_0 L'/(\pi A')] \right]. \quad (5)$$

Here $A'$ is the area of the cross section, which can be arbitrarily convex (circular, rectangular, etc.), $L'$ is the circumference of the cross section, and $u$ and $v$ are both constants:

$$u \approx 3.25/d, \quad v \approx 0.47d. \quad (8)$$

Lastly, the first two terms of the source functions $Q_i$ are formulated as

$$Q_1 = \frac{Q(t)}{4\pi A'} \delta(x-x_0), \quad (9a)$$

$$Q_2 = \frac{Q(t)}{8\pi^2 A'} \delta(x-x_0) \times \int_{0}^{2\pi} u[D(y_0, z_0, \phi) - v] d\gamma, \quad (9b)$$

where $Q(t)$ is the sound source power, $\delta$ is the Dirac function, and $D(y, z, \phi)$ is the distance from an interior point $(x, y, z)$ to the boundary along the direction $-\phi$ (where $\phi$ is the corresponding flight direction of $\gamma$).

Two types of boundary conditions for the opposing ends of the long enclosure are proposed to take the absorption into account, when the reflections are either diffuse or specular. The specularly reflecting boundary condition is

$$\psi_i(0, \mu, t) = R' \psi_i(0, -\mu, t), \quad 0 < \mu \leq 1, \quad (10a)$$

$$\psi_i(L, \mu, t) = R' \psi_i(L, -\mu, t), \quad -1 \leq \mu < 0, \quad (10b)$$

$$\psi_i(0, \mu, t) = R' \psi_i(0, -\mu, t), \quad 0 < \mu \leq 1, \quad (10c)$$

$$\psi_i(L, \mu, t) = R' \psi_i(L, -\mu, t), \quad -1 \leq \mu < 0, \quad (10d)$$

where $R'$ and $R''$ are the reflection coefficients of the two ends, respectively.

The diffusely reflecting boundary conditions as follows:

$$\psi_i(0, \mu, t) = 2R' \int_{-1}^{0} (-\mu') \psi_i(0, \mu', t) d\mu', \quad 0 < \mu \leq 1, \quad (11a)$$

$$\psi_i(L, \mu, t) = 2R'' \int_{0}^{1} \mu' \psi_i(0, \mu', t) d\mu', \quad -1 \leq \mu < 0, \quad (11b)$$

\[ \psi_2(0, \mu, t) = \frac{R' u^2}{2 \pi \mu A'} \left[ \int_A \left( \int_0^{2 \pi} Dd\gamma \right) dydz - 4 \pi^2 v^2 A' \right] \times \int_{-1}^{1} - \mu' \psi_2(0, \mu', t) d\mu', \quad 0 < \mu \leq 1, \]  
\tag{11c} 
\[ \psi_2(L, \mu, t) = \frac{R' u^2}{2 \pi \mu A'} \left[ \int_A \left( \int_0^{2 \pi} Dd\gamma \right) dydz - 4 \pi^2 v^2 A' \right] \times \int_{0}^{1} \mu' \psi_2(0, \mu', t) d\mu', \quad -1 \leq \mu < 0. \]  
\tag{11d} 

Note for the diffusely reflecting boundary condition since

\[ \frac{R' u^2}{2 \pi \mu A'} \left[ \int_A \left( \int_0^{2 \pi} Dd\gamma \right) dydz - 4 \pi^2 v^2 A' \right] \]  
\tag{12}

is usually very small, the present numerical simulations assume it to be zero. Therefore, Eqs. (11c) and (11d) become

\[ \psi_2(0, \mu, t) = 0, \quad 0 < \mu \leq 1, \]  
\tag{13a} 
\[ \psi_2(L, \mu, t) = 0, \quad -1 \leq \mu < 0. \]  
\tag{13b} 

A linear combination of these two boundary condition types in terms of the absorption and scattering coefficients can be used for partially diffuse reflection. However, for simplification, this study only considers purely specular or diffuse reflections.

The sound pressure level is of the most concern and can be written as

\[ L_p(x, y, z, t) = 10 \log \left( \frac{P(x, y, z, t) \rho c}{P_{ref}} \right), \]  
\tag{14} 

where

\[ I(x, y, z, t) = \int_0^{2 \pi} \int_{-1}^{1} \psi d\mu d\gamma \]  
\tag{15} 

is the magnitude of the sound intensity, \( \rho \) is the air density, and \( P_{ref} = 2 \times 10^{-5} \) Pa is the pressure reference.

The transport equation models described above will be most accurate when the enclosure considered is sufficiently long (or narrow), as the unknown functions \( \psi / x, \mu, t \) imply that the sound energy flux is weakly dependent on \( y, z, \text{ and } \gamma \). The limits of aspect ratios (correlated dimensions) where this theory can be applied should be determined on a case-by-case basis in light of the desirable accuracy. However, in this study, the ratio between the length and width (or height) is always larger than 8, and good accuracy has been obtained.

**B. Solution method**

The one-dimensional transport equation model, which consists of integropartial differential equations, can be solved through the discrete ordinate method.\(^{16}\) This study utilizes a finite element modeling software to generate the mesh in the domain using the Galerkin method and solve the equations. The spatial \( x \) and angular \( \mu \) variables are discretized (see Fig. 1). Lagrange-quadratic and Lagrange-linear elements are used (basis functions in the finite element implementation are either linear or quadratic on each mesh interval). Their suitability will be discussed. The mesh condition can be found in the convergence study (Sec. III).

When solving the steady-state transport equation numerically, artificial oscillations arise, primarily due to two reasons: (1) the equation is convection dominated and (2) the point source term in Eq. (3) introduces a discontinuity/singularity, which cannot be well resolved by the mesh. This will cause an initial disturbance that propagates through the computational domain. Refining the mesh resolution does not reduce the numerical oscillations since the discontinuity still exists and there is no diffusion term in the equation. Also it will be overly demanding of computing resources. Therefore, the anisotropic diffusion technology\(^{18–20}\) is employed to suppress the oscillations and guarantee reasonable calculation time. The following simple example will briefly explain this. For more information, the reader is referred to Refs. 18–20. Assuming a convection-diffusion transport equation (a parabolic partial differential equation),

\[ \frac{\partial u'}{\partial t} + \varepsilon \nabla u' = \nabla (\tau \nabla u'). \]  
\tag{16} 

The Péclet number (Pe)\(^{18–20}\) is defined as a function of the diffusion coefficient \( \tau \), convection function \( \varepsilon \), and the mesh element size \( h \),

\[ \text{Pe} = \frac{\|\varepsilon\| h}{2 \tau}. \]  
\tag{17} 

Since the finite element discretization method used is the Galerkin method, solving such a transport equation becomes unstable when the Péclet number (Pe) is larger than 1. Notice that, in the acoustic transport equation [Eq. (3)], there is no diffusion term, implying that the Péclet number is always larger than 1.

By adding an artificial diffusion term \( \tau_{art} \) it is possible to make the Péclet number smaller than 1. The convection term \( \varepsilon \) is a vector, and anisotropic artificial diffusion adds diffusion only in the direction of the streamline. That is, there is no diffusion added in the direction orthogonal to \( \varepsilon \). We now have

\[ \tau_{art} = \frac{Q \varepsilon_i \varepsilon_j}{\|\varepsilon\|^2}, \quad \text{Pe} = \frac{\|\varepsilon\| h}{2(\tau + \tau_{art})}, \]  
\tag{18} 

where \( Q \) is a constant and \( \varepsilon_i \) and \( \varepsilon_j \) are components of \( \varepsilon \). In this way, the Péclet number measured in the streamline di-
The one-dimensional transport equations for steady-state are solved where the time \( t \) is neglected. Numerical results given are from a mesh resolution of 100 angular points and 200 spatial points. Calculation time is around 10 s for the one-group model and 40 s for the two-group model on a current laptop personal computer. The present results are compared with the results from Ref. 16 in Table I, showing good agreements.

### 2. Convergence study

The goal of this section is to first study the convergence condition when numerically solving the steady-state one-dimensional transport equation models and then to briefly compare the transport equation models with the ray-tracing based method, i.e., the solution of the three-dimensional transport equation. This section only discusses the two-group model since it is expected to be more accurate based on the previous discussion.\(^1\) For steady-state transport equation models, the time variable \( t \) is simply discarded. An imaginary long room with dimensions \( 80 \times 4 \times 4 \) m\(^3\) is studied. The scattering coefficient is 0.8. The two ends are considered to be specularly reflecting. The absorption coefficient for all surfaces is 0.5. An omnidirectional source is located at position (0, 0, 0) and the duct is infinite along the \( z \)-direction. The walls are completely absorbing and the sound pressure level at position (60, 2, 2) m for different mesh resolutions. Note that the solution given by the ray-tracing method is 68.3 dB, and the calculation time is more than 5 min for obtaining well converged results. Table II lists the numerical results and suggests that the result in-

### III. NUMERICAL RESULTS

#### A. Steady-state case

##### 1. Verifications

This section discusses verifications of the present solution method. Same as in Ref. 16, where the study is about neutral particle transport, a circular duct with radius 1 m and length 50 m is tested. The boundaries are all diffusely reflecting. Sound particles enter the duct through one end \((x=0)\) of the duct in an isotropic way and leave the duct through the other end which is completely open. The reflection probability at \( x=0 \) is defined as \(^{16}\)

\[
\hat{R} = \frac{\int_0^1 \mu \psi_1(0,\mu) d\mu}{\int_0^1 \mu \psi_1(0,\mu) d\mu}.
\]

The probability of reflection for a circular duct with isotropic incidence.

### TABLE I. Comparison between the present numerical results with the ones in Ref. 16.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( N=1 )</th>
<th>( N=1 )</th>
<th>( N=2 )</th>
<th>( N=2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.018</td>
<td>0.018</td>
<td>0.0255</td>
<td>0.0256</td>
</tr>
<tr>
<td>0.2</td>
<td>0.038</td>
<td>0.038</td>
<td>0.0559</td>
<td>0.0541</td>
</tr>
<tr>
<td>0.3</td>
<td>0.062</td>
<td>0.061</td>
<td>0.0859</td>
<td>0.0862</td>
</tr>
<tr>
<td>0.4</td>
<td>0.089</td>
<td>0.089</td>
<td>0.1225</td>
<td>0.1229</td>
</tr>
<tr>
<td>0.5</td>
<td>0.123</td>
<td>0.122</td>
<td>0.1652</td>
<td>0.1656</td>
</tr>
<tr>
<td>0.6</td>
<td>0.164</td>
<td>0.164</td>
<td>0.2165</td>
<td>0.2170</td>
</tr>
<tr>
<td>0.7</td>
<td>0.219</td>
<td>0.218</td>
<td>0.2808</td>
<td>0.2814</td>
</tr>
<tr>
<td>0.8</td>
<td>0.296</td>
<td>0.295</td>
<td>0.3671</td>
<td>0.3677</td>
</tr>
<tr>
<td>0.9</td>
<td>0.423</td>
<td>0.423</td>
<td>0.5008</td>
<td>0.5014</td>
</tr>
<tr>
<td>0.99</td>
<td>0.751</td>
<td>0.751</td>
<td>0.8024</td>
<td>0.8027</td>
</tr>
</tbody>
</table>

\(^a\)From Ref. 16.
deed converges to a finite value as either the spatial or angular resolution increases. In addition, the difference between each prediction is within a tolerable range, which indicates that for predicting sound pressure level, a coarse mesh can be used for solving the one-dimensional two-group transport equation model. The most accurate result naturally occurs using the finest mesh. This agrees well with the ray-tracing result.

### 3. Comparison with ray-tracing simulations

This section compares numerical results from the present transport equation model with the ones from the ray-tracing based method implemented by a commercially available software CATT-acoustics®, for two representative scenarios. A long room with dimensions $60\times6\times6$ m³ is modeled. An omnidirectional sound source is located at $(5, 3, 3)$ m with sound power of 0.01 W. The receivers are along the line $y=z=3$ m. For the first case, the absorption coefficient is uniform and is 0.05, while in the second case, it is 0.5. The scattering coefficient for all side walls in the first case is 0.9 and 1.0 in the second case. The two end walls are diffusely reflecting. The number of rays for each ray-tracing is 500 000. The truncation time is long enough so that the sound energy decays to a negligible amount. When solving the transport equation, 10 000 mesh elements are constructed. Figures 3 and 4 show the sound pressure level distribution along the $x$-axis across the center of the long room. The following observations are worth discussing.

1. For both examples, the two-group model agrees fairly well with the ray-tracing simulation, while the one-group model agrees less well, especially when the receivers are far away from the source, and the absorption involved is high. This observation is consistent with the conclusion in Larsen’s paper.¹⁶

2. For low absorption case, the maximum deviation of the one-group model from the ray-tracing simulation is around 2.0 dB, which is still acceptable. This suggests that, for stationary sound field predictions in weakly damping long rooms, both one- and two-group models can be employed. Particularly, if the sound field far away from the source is not of major concern, the one-group model works reasonably well.

3. In Fig. 4, all the three curves show that the sound pressure level increases slightly when the receiver approaches the end $(x=60$ m). This important characteristic, however, cannot be captured by the diffusion equation model,²² because the diffusion equation model is not accurate near the boundary.

### B. Time-dependent case

#### 1. Convergence study

This section studies the convergence condition for the time-dependent case in terms of the reverberation time (RT) and early decay time (EDT), again for the two-group model. Compared with the steady-state case, one additional variable, i.e., time $t$, needs to be counted. The choice of the time step can be justified by the following figure.

![FIG. 3. Comparisons of sound pressure level distributions for a long room (60×6×6 m³, uniform absorption coefficient of 0.05, scattering coefficient of 0.9, two end walls are diffusely reflecting) among ray-tracing simulations: one- and two-group models.](image)

![FIG. 4. Comparisons of sound pressure level distributions for a long room (60×6×6 m³, uniform absorption coefficient of 0.5, scattering coefficient of 1.0 for all the walls) among ray-tracing simulations: one- and two-group models.](image)

### Table II: Sound pressure level prediction with varying mesh resolutions for a long room.

<table>
<thead>
<tr>
<th>Spatial point number</th>
<th>Angular point number</th>
<th>Sound pressure level (dB)</th>
<th>Calculation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>20</td>
<td>68.38</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>68.55</td>
<td>4</td>
</tr>
<tr>
<td>50</td>
<td>100</td>
<td>68.62</td>
<td>9</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>68.20</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>68.37</td>
<td>8</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>68.44</td>
<td>17</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>68.11</td>
<td>6</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>68.29</td>
<td>15</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>68.36</td>
<td>35</td>
</tr>
<tr>
<td>400</td>
<td>20</td>
<td>68.07</td>
<td>9</td>
</tr>
<tr>
<td>400</td>
<td>50</td>
<td>68.24</td>
<td>31</td>
</tr>
<tr>
<td>400</td>
<td>100</td>
<td>68.32</td>
<td>72</td>
</tr>
</tbody>
</table>

![TABLE II. Sound pressure level prediction with varying mesh resolutions for a long room.](image)
determines the accuracy of the solution and the calculation time, and therefore needs to be carefully examined. For results summarized here, the total number of mesh elements is changed, i.e., the product of both distance and angular point numbers, rather than both distance and angular mesh sizes being varied independently. Two types of mesh elements are considered: quadratic and linear. As anticipated, the former requires longer calculation time. The decay time parameters are found by exciting an impulse in the room using Eq. (20), solving for the sound intensity impulse response from the transport model, and using Schroeder integration to compute for the sound intensity decay function. For calculating the reverberation time, the normalized decay level interval of −5 to −35 dB is used, and for early decay time, from 0 to −10 dB.

The long room model has the same geometry, with dimensions $80 \times 4 \times 4 \text{ m}^3$, as that for the steady-state convergence investigation, the source is also the same, the receiver is located at $(20, 2, 2) \text{ m}$, the scattering coefficient for the side walls is 0.8, and the absorption coefficient for all walls is 0.4. For a narrow long room, if the two ends are specularly reflecting and strongly reflective, a randomized ray that happens to shoot exactly perpendicular to the end walls will continue to bounce back and forth between the end walls and create flutter echoes, which makes the energy decay curve irregular/nonexponential. In addition, relatively strong numerical damping has been found for the sound traveling in the direction perpendicular to the end walls ($\mu = \pm 1$). Therefore, in the time-dependent case, only diffusely reflecting ends are included in the discussions. The total calculated time length for the transport equation model is 0.8 s, which is much longer than the expected reverberation time. The simulation results given by ray-tracing are 0.38 and 0.35 s for reverberation and early decay times, respectively. The calculation by ray-tracing takes more than 15 min. Figure 5 shows the energy impulse response, energy-time curve, and Schroeder integration curve at the receiver position. The transport equation model simulates the direct sound properly as the energy peak shown approximately at 0.06 s (the time the source signal arrives at the receiver) within one calculation run. The group model shows noticeable discrepancies with the ray-tracing results for reverberation times. All the models are capable of predicting sound energy decay for all the side walls is 0.7. The two end walls are diffusely reflecting, i.e., $\mu = 1.0$. The number of rays is 500,000 for the ray-tracing implementation. 8000 quadratic elements are generated when solving the transport equation. The time step is 0.001 s. Figure 6 shows the reverberation and early decay time trends along the $x$-axis. The two-group transport model agrees very well with the ray-tracing results, while the one-group model shows noticeable discrepancies with the ray-tracing results for reverberation times. All the models are able to show that reverberation time increases slowly while early decay time increases quickly.

In the second example, the absorption coefficients are 0.4 for the side walls and 0.01 for the two end walls. The scattering coefficient is reduced to 0.5. Figure 7 shows the reverberation and early decay time results. Note that the one-group model is not included in this comparison. The discrepancy between the ray-tracing results and transport equation model results is still sufficiently small. These two cases suggest that the transport equation model, especially the two-group model, is fully capable of predicting sound energy decays in a long space. At least the accuracy of the present model is on the same level as a commercial acoustic prediction software.

IV. EXPERIMENTAL RESULTS

A. Experimental setup

A long room tenth scale-model is built to further assess the accuracy of the transport equation model. The reasons for...
choosing physical scale modeling are its versatility and easy access.\textsuperscript{10,25} The dimensions of the scale-model are $2.4 \times 0.24 \times 0.24$ $m^3$, which corresponds to $24 \times 2.4 \times 2.4$ $m^3$ for real size. This scale-model is built of $\frac{3}{8}$ in.-thick hard plywood. The wall surfaces are relatively smooth, only featured with small scale roughness ($<0.2$ $cm$ in depth). Figure 8 shows a photograph of the scale-model with the top and two ends open. A miniature dodecahedron loudspeaker system used as the sound source is located at $(6.1, 1.2, 1.2)$ $m$ [see Fig. 9(a)]. It is reasonably omnidirectional up to $32$ kHz. A $\frac{1}{4}$ in. microphone is used as the receiver to measure the room impulse responses excited by maximal-length sequences of $2^{18} - 1$, averaged over ten repetitions. The microphone is moved along the centerline of the scale-model throughout the measurements. Fourteen measurements are conducted with an equidistant separation along the $x$-axis and are repeated five times to show the repeatability/uncertainty of the experiment. Measurements of the sound field close to the source are avoided since the source directivity has a greater impact on the near field. Neither nitrogen nor air drying is used to compensate for the air attenuation, as the air attenuation can be included in the transport equation model (see Sec. IV B), and the purpose of this measurement is only to verify the present model rather than to model an existing space. (The following discussion will only use real-sized dimensions.)

### B. Experimental verifications of the transport equation model

This section aims to verify the transport equation model by comparing the experimental results with the simulation results. As the geometrical-acoustic model works only properly in a high frequency broad-band range, the results at 1–2

<table>
<thead>
<tr>
<th>Element No.</th>
<th>Element type</th>
<th>Time step (s)</th>
<th>RT (s)</th>
<th>EDT (s)</th>
<th>Calculation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>Quadratic</td>
<td>0.002</td>
<td>0.404</td>
<td>0.419</td>
<td>19</td>
</tr>
<tr>
<td>1000</td>
<td>Quadratic</td>
<td>0.001</td>
<td>0.406</td>
<td>0.419</td>
<td>30</td>
</tr>
<tr>
<td>2000</td>
<td>Quadratic</td>
<td>0.0005</td>
<td>0.406</td>
<td>0.418</td>
<td>53</td>
</tr>
<tr>
<td>2000</td>
<td>Linear</td>
<td>0.002</td>
<td>0.401</td>
<td>0.376</td>
<td>38</td>
</tr>
<tr>
<td>2000</td>
<td>Quadratic</td>
<td>0.001</td>
<td>0.402</td>
<td>0.379</td>
<td>61</td>
</tr>
<tr>
<td>2000</td>
<td>Linear</td>
<td>0.001</td>
<td>0.001</td>
<td>0.019</td>
<td>18</td>
</tr>
<tr>
<td>2000</td>
<td>Quadratic</td>
<td>0.0005</td>
<td>0.403</td>
<td>0.380</td>
<td>105</td>
</tr>
<tr>
<td>2000</td>
<td>Linear</td>
<td>0.0005</td>
<td>0.001</td>
<td>0.019</td>
<td>18</td>
</tr>
<tr>
<td>5000</td>
<td>Quadratic</td>
<td>0.002</td>
<td>0.401</td>
<td>0.360</td>
<td>99</td>
</tr>
<tr>
<td>5000</td>
<td>Linear</td>
<td>0.002</td>
<td>0.391</td>
<td>0.374</td>
<td>17</td>
</tr>
<tr>
<td>5000</td>
<td>Quadratic</td>
<td>0.001</td>
<td>0.404</td>
<td>0.362</td>
<td>166</td>
</tr>
<tr>
<td>5000</td>
<td>Linear</td>
<td>0.001</td>
<td>0.387</td>
<td>0.376</td>
<td>29</td>
</tr>
<tr>
<td>5000</td>
<td>Quadratic</td>
<td>0.0005</td>
<td>0.404</td>
<td>0.360</td>
<td>273</td>
</tr>
<tr>
<td>5000</td>
<td>Linear</td>
<td>0.0005</td>
<td>0.387</td>
<td>0.377</td>
<td>52</td>
</tr>
</tbody>
</table>
kHz octave broadband are chosen for comparison. In lower frequency bands, wave phenomena, e.g., standing waves, are observed, which cannot be modeled by the transport equation model. In the frequency range of interest (1–2 kHz), the absorption coefficient is estimated to be 0.15. The scattering coefficient is difficult to estimate but is expected to be low (as mentioned above, the wall surface is relatively smooth). An average air attenuation constant of 0.00391 Np/m is used for 10–20 kHz for the tenth-scale factor (temperature of 20 °C and humidity of 50%).

This section only considers the two-group model. The reflections at the two ends are set to be completely diffused. Since two ends of the scale-model are relatively smooth, this will introduce a source of error; however, it is expected to be insignificant because the two ends are small compared with the side walls. For both steady-state and time-dependent cases, two simulations with scattering coefficients of 0.1 and 0.3, respectively, are carried out.

Figure 9(b) shows the comparison of sound pressure level distributions. The transport equation model with scattering coefficient of 0.3 agrees well with the experimental results. Most of results show small deviations (<1 dB). The transport equation model with scattering coefficient of 0.1 matches the experimental results less well but the disparity is still tolerable. This implies that the scattering coefficient is indeed low. To further confirm this, the simulated and measured reverberation times will be compared. Figure 10(a) presents a segment of the room impulse responses recorded 4.6 m from the source. Figure 10(b) illustrates the Schroeder curves given by experimental measurement in comparison with the transport equation model with scattering coefficients of 0.1 and 0.3.

Figure 11 shows a detailed comparison of the reverberation time prediction. Both simulation curves agree well with the measurement. Together with the sound pressure level distribution results, it appears that the scattering coefficient is likely to be between 0.1 and 0.3. So far, the experimental comparisons have demonstrated the validity of the one-dimensional transport equation model for a long space.
V. CONCLUSIONS

This paper has discussed numerical results for a long space room-acoustic model based on one-dimensional transport equations. The transport equation models explicitly handle partial specular, partial scattering, tolerate high overall absorption, and contain direct sound and early reflection portions of sound propagation. Comparisons between the transport equation model and the ray-tracing approach indicate that the solutions given by the one-dimensional transport equation model well approximate the three-dimensional exact solution. Particularly, the two-group model can work properly in a wide range of scenarios. In addition, numerically solving the transport equation models is significantly less time consuming than the ray-tracing approach. Note that the current solution method for the transport equation models is not specifically optimized; therefore, it is possible to speed up the simulation by using advanced technologies. Finally, experimental results from a long room scale-model further validate the transport equation model.

Within the scope of this work, only empty rooms and omnidirectional sound sources are considered. However, these transport equation models can also take interior scattering objects and source directivity into account. These transport equation models can be extended to simulate side walls with nonuniform absorption or scattering coefficients, which makes the model more flexible. This study points out that the transport equation model is more accurate than the diffusion equation model, e.g., near a boundary. Detailed comparisons between the transport and diffusion equation models, which will clarify the relationship of these two models, are also attractive. These issues are expected to be addressed in future work.

ACKNOWLEDGMENTS

The authors would like to thank Professor E. Larson, Professor H. Kuttruff, Profesor W. Siegmann, and Professor J. Wei for their helpful discussions. The authors would also like to thank Mr. Joon Hee Lee and Mr. Gino Pellicano for their assistance in collecting experimental results, and Mr. Robert Bocala and Dr. Phillip Jason White for their critical reviews of the manuscript.

11P. M. Morse and H. Feshbach, Methods of Theoretical Physics (McGraw-Hill, New York, 1953).