Introduction
Sound quality of concert halls and auditoriums is a matter of taste because there are as yet no overall objective measurable criteria that describe the quality of acoustics in the room. Instead, sound quality is a subjective measure that is often determined by listening in the space. For this reason, it is essential that the acoustic designers of concert halls and auditoriums get a chance to listen to the expected outcome as early as possible, preferably even before construction has started (Hochgraf, 2019).

Providing such an opportunity is the goal of simulation-based auralization, which aims to produce as authentic an auditory experience as possible by utilizing only the geometry and acoustic treatment information of the space. Overall, the main ingredients of room-acoustic auralization are room-impulse responses, anechoic recordings, and spatial sound reproduction systems, as illustrated in Figure 1. The room-impulse responses can be obtained in terms of either simulation or measurement. The measurement-based auralization is applicable when an existing space or a scale model is available to be auralized. In this paper, we focus on different room-acoustic modeling techniques and their use in simulation-based modeling and binaural auralization. Other approaches, including measurement-based (Schroeder, 1970; Xiang and Blauert, 1993) and multichannel (Blauert and Rabenstein, 2017) auralization are out of our current scope.

Computational modeling and auralization in room acoustics were conceived in the early 1960s when Schroeder et al. (1962) presented the basic ideas. In its early stage, room-acoustic computer simulation was developed mostly without audible components (Krokstad et al., 1968; Schroeder, 1970).

Figure 1. Room acoustic auralization can be based either on simulations or measurements, and the results can be made audible via headphones or loudspeakers. The main component of auralization is the convolution of room-impulse responses with anechoic signals.
Auralization
Motivated by "visualization" as used for visual rendering, Kleiner et al. (1993) coined the term "auralization" in the context of room-acoustic modeling. They did this when they reviewed research activities in the field of room-acoustic simulations in the early 1990s, including computer-aided modeling in the form of both numerical simulation and experimental measurements in physical models. Summers (2008) notes "that auralization represents acoustic modeling as the agent that performs the rendering for the purpose of auditory perception and the sound events being rendered are created via simulation."

In an overview of reverberation techniques, Välimäki et al. (2012) convey the idea that the focus of room-acoustic modeling is to obtain room responses by computational simulation, whereas Xiang and Blauert (1993) developed binaural auralization using binaural measurements in physical scale models. Those scale-modeled responses are processed, leading to binaural samples that can then be rendered for auditory perception. Vorländer (2008) also uses the term auralization to encompass any process that yields sound samples through modeling, synthesis, or experimental measurements. Therefore, auralization collectively encompasses both the modeling process and their results (Summers, 2008). In recent years, auralization has become an effective design tool to support acoustic designers in their innovative designs (Hochgraf, 2019). In this paper, we make a further distinction between simulation-based and measurement-based auralization and concentrate on the simulation part.

Linear time-invariant systems are often used to describe sound transmission from sound sources to receivers in room-acoustic enclosures. The transmission from one sound source to one monophonic receiver within a space is fully described by the one-room impulse response (RIR). A linear convolution of the RIR with sound signals recorded in an anechoic environment, being free from any reflections, yields sound samples as if the sound travels from the sound source in the space and arrives at the monophonic microphone.

For a binaural-listening situation, binaural RIRs characterize sound traveling through the room when they arrive at the listener’s two ears (Xiang and Blauert, 1993). The linear convolution of the binaural RIRs with reflection-free sound samples leads to one pair of binaural sound samples. This convolution is typically implemented in the frequency domain using fast Fourier transform. When these sound signals are displayed properly to the two ears of the listener, the listener will perceive auditory scenes as if the individual were sitting inside the enclosure. The binaural auralization via room-acoustic modeling and the virtual auditory reality are evolved from this fundamental principle (Vorländer, 2020).

Crucial to the binaural auralization are the directional and spectral properties of the binaural receiver involved in the modeling. The external ears of a listener, consisting of two pinnae, the head, and the torso, encode spatial information of the incident sound field into only two channel signals in specific filtering. Head-related transfer functions (HRTFs) represent the transfer functions of the filtering in the frequency domain. In the time domain, they are known as head-related impulse responses (Blauert, 1997). They can be obtained in the form of a databank established in the early days through extensive measurements (Gardner and Martin, 1995) and later by numerical simulations such as applying the finite-difference time-domain approach. With advanced optical scanning of three-dimensional (3D) objects such as the

Figure 2. Three-dimensionally (3D) printed models using the meshing data of an artificial head for validation against the original grid mesh around a popular artificial head with pinnae. The original mesh was created for finite-difference time-domain simulations of head-related impulse responses. Distance errors between the original mesh and the 3D printed head replica (referred to as the Hausdorff error) are largely less than 1 mm, indicated by a pseudocolor scale, which is of sufficient accuracy to represent the pinnae and the head. Reproduced from Prepelita, et al., 2020, with permission.
ear pinnae, head, and even the torso, individual subjects can be conveniently scanned to establish finite-difference models for wave-based simulation of the head-related impulse responses (Prepelita et al., 2020). Figure 2 illustrates a 3D printed head replica using the so-created mesh model to valid meshing accuracy.

**Room-Acoustic Modeling Techniques**

The aim of the room-acoustic modeling is to compute RIRs in the given space as accurately and as efficiently as possible under given constraints. These constraints depend on the needs of the application, available computational resources, and so on. The range of applications is wide, starting from the acoustic design of concert halls (Hochgraf, 2019) where accuracy is crucial and computational performance is only secondary to room-acoustic research, and ending in computer games where the situation is completely opposite and real-time performance is required at the cost of low accuracy (Raghuvanshi et al., 2007).

There are many different modeling paradigms, each having their own pros and cons. But what is common to them is that they aim to produce RIRs. Figure 3 schematically depicts an energy RIR, a so-called echogram in a room. In the early part, the energy RIR encompasses direct sound and early reflections, followed by a late reverberation part. We now discuss the main modeling principles and techniques (briefly summarized in Table 1).

**Wave-Based Modeling**

The wave-based modeling techniques are the most accurate models, although they are also the most inefficient because their computational loads typically depend on the frequency range to be covered. As the frequency goes higher, more computational resources are needed. As a result, they are computationally extremely expensive at the high end.

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**Table 1. Capabilities and properties of two room-acoustic modeling techniques**

<table>
<thead>
<tr>
<th>Reflection Type</th>
<th>Wave Based</th>
<th>Geometrical Acoustic</th>
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<tbody>
<tr>
<td></td>
<td>Energy/pressure</td>
<td>Pressure</td>
</tr>
<tr>
<td>Specular</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Diffuse</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Edge diffraction</td>
<td>Inherent</td>
<td>Extension</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Main factor</th>
<th>Grid density</th>
<th>N/A (is exact)</th>
<th>Number of rays</th>
<th>Grid density</th>
<th>Grid density</th>
</tr>
</thead>
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<table>
<thead>
<tr>
<th>Computational Load</th>
<th>wrt Frequency</th>
<th>Polynomial</th>
<th>Constant</th>
<th>Constant</th>
<th>Constant</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>wrt Time</td>
<td>Linear</td>
<td>Exponential</td>
<td>Linear</td>
<td>&gt;Linear</td>
<td>Linear</td>
</tr>
</tbody>
</table>

FDTD, finite-difference time-domain method; FEM, finite-element method; BEM, boundary-element method; wrt, with respect to. The varying capabilities and properties depend on the modeled quantities (pressure or energy), on the capability of involving scattering or other phenomena, and on the main factors affecting their accuracy and computational complexity.
The wave-based approaches naturally capture all wave phenomena in enclosures, including interference and diffraction, and they allow for a detailed modeling of the interior boundary. Solution of the acoustic-wave equation represents the most strict modeling, a second-order partial differential equation that characterizes the wave propagation or its frequency-domain version, the Helmholtz equation. Solving these equations is challenging, particularly for complex geometries.

Practical wave-based solvers, therefore, apply some discrete grids of the space and/or time. These methods include the finite-difference/finite-volume time-domain (FDTD/FVTD), the time-domain spectral element (TD-SEM), the finite-element (FEM), and the boundary-element (BEM) methods. Botteldooren (1994) first applied the FDTD in room-acoustic simulations, and there has been significant progress in the ensuing years, such as on how to model complex geometries (Bilbao, 2013). Pind et al. (2019) investigated an attractive TD-SEM approach with geometrical flexibility because it is also able to incorporate a complex-valued frequency-dependent boundary. The recent special issue of *The Journal of the Acoustical Society of America* (Savioja and Xiang, 2019) reported the latest progress in incorporating source directivities in FDTD simulations, which involves the perceptual study of inherent dispersion errors specific in FDTD modeling. The special issue also includes an approach using the discontinuous Galerkin method, further development of the FEM incorporating source, and receiver directivity for auralization.

These techniques typically discretize the space into a grid, and the density of this grid determines the bandwidth that can be simulated. After that, the various techniques compute the solution iteratively for the whole grid. For auralization purposes, it is beneficial to store all these results, and while moving, use some interpolation between responses from different grid points such that the most correct response for any receiver location is obtained. Note that the FDTD and FEM use a volumetric grid, whereas the BEM utilizes a surface grid such that responses are stored at surfaces and can then be gathered and integrated to a given location. This is a straightforward operation for monophonic responses, but for spatial reproduction, the situation is more complex because it requires storing some spatial information instead of monophonic responses.

**Geometrical-Acoustic Room Simulations**

Since the 1960s, room-acoustic modeling, in principle, has employed geometrical acoustics. The geometrical acoustics basically assumes that sound waves propagate along straight lines like light rays, and all the wave phenomena, such as diffraction and interference, are neglected (Savioja and Svensson, 2015). Computational simulations based on the geometrical acoustics are highly efficient but less accurate when compared with the wave-based models. These geometrical-acoustic methods have been in room acoustic practice and research for over 50 years since Krokstad et al. (1968) published their seminal work on acoustic ray tracing. Another key method in the geometrical-acoustic regimen is the image-source method. Although it was applied in room acoustics as early as Eyring (1930), the widespread adaptation of the image-source method is attributable to Allen and Berkley (1979).

Geometrical room-acoustic simulations did not include an auralization capability until Pösselt (1987) incorporated the head-related impulse responses into image source-based room simulations, the end results of which were binaural audible samples that could be rendered using a set of headphones (see also Blauert and Pösselt, 1988). Pösselt’s (1987) pioneer modeling effort, although in a rectangular room, opened up opportunities for computer simulations (first based on a geometrical-acoustic principle) to create a computer model of the sonic environment for a listener as if (s)he were sitting in the simulated space, listening the sound field using her/his own ears. Blauert et al. (1990) provided a brief review on *binaural room simulation*. Binaural room simulation itself includes binaural rendition for auditory perception through acoustic room simulations. A stream of research activities on both fundamental and application levels of auralization followed, leading to a boom in room acoustic modeling and auralization research as illustrated in a special issue of *Applied Acoustics* (Naylor, 1993).

In **Image-Source Methods**, we present the fundamental concepts underlying key geometrical acoustics methods. The techniques presented heavily rely on an overview by Savioja and Svensson (2015).

**Image-Source Methods**

The image-source method recursively constructs the image sources to the sound source in the room. A sound source is image reflected against all interior surfaces,
resulting in secondary image sources that are again image reflected against all the surfaces. This represents a recursive process until termination criteria are met, such as reflection order or energy threshold. The resulting image sources are essentially secondary sources, each of which carries a reflection history. The distance from the image source to a receiver is used to determine the actual reflection path length. Figure 4 illustrates schematically this recursive construction of the image sources in a two-dimensional case. Four first-order reflections along with six second-order reflections are illustrated (actually there are eight second-order ones). Generally, the number of image sources up to $M$th-order reflections is given by $\sum_{i=1}^{M} N(N-1)^{i-1}$ for $N$ surfaces. The image-source method yields exact solutions for rectangular rooms and ideally hard surfaces, although in real rooms, there is always some modeling error due to real material properties and lacking modeling of edge diffraction.

Borish (1984) extended the image-source method for arbitrary room shapes, allowing a high flexibility of geometry. The recursive construction of reflections is the same as that of rectangular rooms, yet additional checks need to be pursued. For example, if the previous reflector completely covers the current reflector, there is no need to create a new image source. All of these image sources are independently computed from the receiver position. Thus, the resulting image sources are valid for the entire room. From the viewpoint of the auralization, this means, that all the image sources can be precomputed for a given sound source.

The last step is checking the “visibility” of each image source to a given listener location. This step needs to be repeated whenever the receiver moves, and it creates a specular reflection path from the source to the receiver, as illustrated in Figure 5a. To pass the test, the path must reach all the reflecting surfaces within the room boundaries and may not be obstructed by any other surface. The sum of the contributions of all visible image sources leads to the room impulse response. For binaural auralization, a binaural receiver in the form of a HRTF is straightforwardly incorporated, resulting in one pair of binaural room impulse responses. The source directivities can be incorporated by image reflecting these source directives along with the locations (Savioja et al., 1999).

The computational load of the image-source technique grows exponentially with the number of image sources,
but it computes early reflections with high efficiency. Therefore, it is often applied in virtual auditory reality to facilitate real-time auralization (Savioja et al., 1999; Vorländer, 2020), whereas at higher order reflections, the technique quickly becomes intractable.

In a typical room geometry, most of the image sources provided by the image-source method are obstructed such that they can never became audible and thus cause lots of unnecessary computation. Funkhouser et al. (2004) developed an image source-based beam tracing that takes this into account, resulting in high computational efficiency. This is achieved by taking into account the reflecting geometry when new image sources are generated and the only reflections taken into account are those that can produce a visible image source to some receiver location.

**Ray-Tracing Methods**

Ray tracing (Krokstad et al., 1968), which involves the tracing of sound particles (phonons) traveling at sound speed in enclosures like light rays, represents one major method in room-acoustic computer simulation. Figure 5b illustrates the core of ray tracing. A sound source radiates rays that are traced for each reflection and then registered for valid paths. The source radiates rays using either a predefined distribution or in random directions (Krokstad et al. 1968). A known directivity of the source will weigh the ray distribution.

Reflection paths originating from the source and reaching the receiver are determined using detectors that intersect with the rays. The detectors are typically volumetric objects such as spheres so that they register enough rays to give reliable results (Vorländer, 1989). Use of more rays warrants use of smaller detectors and more accurate results. Another option is to use point-like detectors and volumetric rays, typically of conical or of pyramidal shape.

The ray termination and the energy attenuation of sound propagation rely on each other and can be calculated using different approaches. In a common approach, each ray carries its energy content in given frequency bands on an interior surface reflection. The material properties of the surface determines the energy attenuation. The ray termination is eventually determined for its energy to decay below a preselected threshold or to reach a predefined maximum traveling distance.

In practice, one simulation containing multiple receivers is often computationally advantageous. Each such receiver registers an energy room impulse response, as the one schematically illustrated in Figure 3. Note that the convolution operation applied in auralization requires pressure impulse responses where an energy impulse response needs to be further processed to create a room impulse response. In addition, some approximation techniques need to be involved to achieve as realistic outcome as possible (Kuttruff, 1993).

In comparison with the image-source method, the ability to incorporate diffuse reflections represents one unique feature of ray tracing. Krokstad et al. (1968) and Schroeder (1973) discussed the basic concept of incorporating diffuse reflections into ray tracing, but it was Kuttruff (1971) who first implemented ideally diffuse reflections. A more general method engages a specular reflection component and the other diffuse components; a scattering coefficient determines the ratio of the two components. Comparison studies demonstrate that the ray-tracing technique exhibits a superior performance over the image-source method.

**Surface-Based Modeling**

The techniques of this kind first engage a sound source to propagate sound energy to the interior surfaces. Subsequently, the energy is further propagated between surfaces until reaching the receiver, and it can be considered as intensity-based boundary-element methods (BEMs). One approach, so-called acoustic radiosity, only accepts ideally diffuse reflections, whereas the path-based image-source technique only incorporates ideally specular reflections. Kuttruff (1971) presented the basic theory for the acoustic radiosity method. The technique is able to simulate the sound propagation in nonrectangular rooms with ideally diffuse surfaces. This is a multipass technique in which much of the simulation can be computed independently from the receiver. Only the last pass considers the sound energy traveling to the receiver, and so it is especially attractive in interactive simulations where the actual interaction needs to be instantaneously determined based on precalculations of all previous passes. The technique is extremely attractive to real-time auralization.

The downside to this technique, however, is the degraded accuracy of an exact reflection path (Savioja and
Svensson, 2015). Acoustic radiance transfer is a further development of basic radiosity incorporating arbitrary reflection characteristics into the model, lifting the original limitation of only ideally diffuse reflections (Siltanen et al., 2007). The result of these simulations is energy RIRs at surfaces that can all be precomputed. In the interactive simulation, the responses to a given receiver are gathered and processed for auralization similarly as with BEM.

Modeling Based on Transport Theory

Sound particles propagating in enclosed spaces can be described by the transport equation, with field quantities being sound energy density and energy flux. The particles propagate along straight lines and strike partially absorptive walls or objects that also partially scatter the incoming particles. Therefore, the transport-equation modeling is still classified under geometrical acoustics. In room acoustics, Jing and Xiang (2010) seem to be the first to have solved this transport equation for simulating a long space and also to experimentally validate their solutions. They demonstrate both mathematically and experimentally that the transport-equation model is capable of incorporating arbitrary wall properties, including absorption, specular, and diffuse reflection (see Table 1).

Navarro et al. (2010) independently explained exactly the same transport theory in a comprehensible manner, calling it the radiactive transfer model. The transport equation can be simplified asymptotically to the diffusion equation. Room-acoustic modeling using the diffusion equation was first reported by Valeau et al. (2006). Jing and Xiang (2008) proposed a rigorous boundary condition, making the diffusion equation applicable in broader room-acoustic conditions. The recent decade has witnessed a stream of room-acoustic applications using the diffusion equation, partially reflected also in the recent special issue (Savioja and Xiang, 2019).

One of attractive features of the diffusion equation lies in either finite-element or finite-difference implementation, yet the mean-free path length of the space under consideration, rather than wavelengths, primarily dictates the mashing condition in enclosures of proportionated dimension. The simulation can, therefore, be implemented extremely efficiently. Another important feature is that room simulations based on the diffusion equation allow for outputting sound energy flux without extra computational expense due to its deep root in Fick's law (Jing and Xiang, 2008).

Hybrid Models

From the perceptual viewpoint of a listener, the direct sound along with the early reflections, as illustrated in Figure 3, are the most important ones, and so the modeling of the early reflections deserve more attention than detailed modeling of the late reverberation. Some modeling techniques are at their best in accurately modeling accurately this early part, whereas some other techniques can efficiently model the later part. This suggests that a hybrid model combining different techniques can provide an optimal solution. In practice, it is advantageous to use a hybrid technique in the time domain. The technique separately calculates the direct sound and the early reflections by the image-source method, even in real time, and the late part is gathered from precomputed responses or exploiting its random nature by artificial approximation (Xiang et al. 2019), enabling real-time auralization.

Similar division also takes place in the frequency domain. The wave-based models excel at the low frequencies, whereas the geometrical-acoustic models are better suited for higher frequencies. Basically, the wave-based models provide an accurate solution, but their computational load gets excessive at higher frequencies, and for this reason, a somewhat less accurate but more efficient model is better suited for that range.

One additional phenomenon worth consideration is air absorption. In practice, air acts as a low-pass filter, causing higher frequencies to be attenuated much more than the lower range as a function of propagation distance. Modeling this by wave-based solvers increases their computational load, and it is often advantageous to switch to geometrical acoustic models on frequencies in which the air absorption is notable. In those energy-based models, air absorption is straightforward to implement as postprocessing.

Concluding Remarks

Initially conducted in the 1960s, room-acoustic simulations have been progressed remarkably. New modeling paradigms have emerged, and the old ones have been developed to be more efficient. They are capable of highly complex geometries and boundaries.
Auralization has become a powerful tool in architectural acoustics practice, research, modern computer games, and other virtual environments to enhance immersion in virtual worlds. In addition, improved understanding of psychoacoustics has enabled focusing of computational resources to the perceptually most relevant parts of the room-acoustic responses. Recent introduction of massively parallel computation via graphics-processing units has significantly speeded up computation to accomplish real-time simulation and auralization. Despite all this progress, there are still plenty of open research questions; therefore, the room-acoustic simulation still remains an active field of research.

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