Simultaneous acoustic channel measurement via maximal-length-related sequences[a)]

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A wide variety of acoustic systems has multiple sources and receivers. This paper proposes a technique for making acoustic measurements simultaneously for multiple sources. The proposed technique features a collection of excitation signals of the maximum-length sequence (MLS) and MLS-related classes. Each signal in the set has a pulse-like autocorrelation function, and the cross-correlation functions between arbitrary pairs of signals drawn from the set have peak values that are significantly lower than the peak magnitude of the autocorrelation functions. The proposed method is particularly valuable when characterization of multisource, multireceiver system must be accomplished in a limited time period. Both simulation and experimental results are presented that demonstrate the feasibility and fidelity of the proposed techniques in characterizing acoustic systems. © *2005 Acoustical Society of America.* [DOI: 10.1121/1.1868252]

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I. INTRODUCTION

A previous paper[1] presented a technique using reciprocal pairs of maximal-length sequences (MLSs) in simultaneous dual-source channel measurements. Impulse responses between two separate sources and one or several receivers of acoustic systems can be determined simultaneously. The simultaneous dual-source measurements exploit excellent cross-correlation properties of reciprocal MLS pairs. Once the number of simultaneous sources is higher than two, straightforward use of reciprocal MLS pairs cannot meet the need. Special attention must then be given to generation of a larger set of source signals that has acceptable properties for performing the simultaneous measurements.

Because of their favorable cross-correlation properties, certain sets of MLSs and MLS-related sequences have gained considerable attention in spread spectrum communication systems.[2] They are, however, not yet widely applied to acoustic systems, particularly in the area of simultaneous acoustic measurements. Recent research in sound propagation through atmosphere[3,4] particularly calls for the simultaneous measurement technique. For this purpose, we propose here the application of a set of the MLSs and MLS-related classes in which all sequences of the set possess a *pulse-like* periodic autocorrelation function, while the periodic cross-correlation function between any pair of sequences drawn from the set has a peak value that is significantly lower than the peak value of the autocorrelation function. In general, the proposed technique is particularly valuable where multisource, multireceiver system characterization tasks must be accomplished in a limited time period.

In Sec. II we discuss the desired correlation properties of acoustic excitation signals that would be needed to accomplish simultaneous multiple source measurements in an ideal case. We then briefly describe suitable sets of acoustic signals based on binary MLSs and MLS-related sequences. This briefing relies heavily on the literature in the spread spectrum communication.[2,5,6] Using these sets, simultaneous channel characterization can be accomplished to a suitable degree of accuracy. In Sec. III we present simulation and field measurements that demonstrate the feasibility of simultaneous channel characterization. Finally, we draw some conclusions in Sec. IV.

II. CORRELATION PROPERTIES OF MLS AND RELATED SEQUENCES

For simultaneous multiple acoustic source measurements (SMASM), let n and p denote the number of sources and receivers, respectively. Then, the p-element vector of received signals, $\mathbf{r}$, can be obtained from

$$\mathbf{r}=\mathbf{h}*\mathbf{s}, \tag{1}$$

where $\mathbf{s}$ is the $n\times 1$ source signal vector, $\mathbf{h}=[h_{ij}]$ is the impulse response matrix of dimension $p\times n$, and "$*$" denotes the matrix periodic (circular) convolution operator. Each element of $\mathbf{r}$, $\mathbf{h}$, and $\mathbf{s}$ is, again, a discrete function of time, but time dependency is suppressed for brevity. If the periodic cross-correlation function (PCCF) of excitation signals is

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$$\mathbf{s} \otimes \mathbf{s}^{\mathbf{T}} = [\delta], \tag{2}$$

where the operator $\otimes$ stands for the cross correlation, $[\delta]$ denotes the matrix each of whose diagonal elements is a discrete periodic delta function and each of whose off-diagonal elements are the all-zero function, then the impulse response matrix can be directly determined from[1]

$$\mathbf{h} = \mathbf{r} \otimes \mathbf{s}^{\mathbf{T}}. \tag{3}$$

Equation (3) indicates that the individual impulse response sequences, h_{ij}, between source j and receiver i, can be conveniently determined by cross correlating each received signal with each source signal provided the source signals possess the desired correlation property expressed in Eq. (2). Lüke[5] pointed out, however, that signals that exactly fulfill this condition [Eq. (2)] neither exist nor can be constructed. Nevertheless, a careful selection or combination among MLSs of the same degree may yield a set of sequences approximately possessing the desired correlation property.

An n-stage linear feedback shift-register device can generate a binary MLS, $\{a(i)\}$, with a period of $L = 2^n - 1$. The positive integer, n, is said to be the degree of the MLS.[7,8] The sequence obtained through correlation of one bipolar MLS $\{m(i)\}$ with $m_i = 1 - 2a_i$ or MLS-related sequence with another of the same degree, or cross correlation, is of practical significance for the SMASM technique, which has been known in the communication community for many years. They are, however, considerably less widely known in the acoustics community; therefore, a brief description of cross-correlation properties of various sets of sequences drawn from MLS and MLS-related classes is sufficient in the following.

A. Preferred MLS pairs and maximum connected sets

The sequence $\{b(i)\}$, where $b(i) = a(di)$ and d is appropriately chosen, is itself an MLS, known as a *factor-d decimation of the sequence* $\{a(i)\}$,[6] the cross correlation of the sequences $\{b(i)\}$ and $\{a(i)\}$, will yield very small values. If the degree of an MLS is a multiple of 4, then a decimation factor[9] $d(n) = 2^{(n+2)/2} - 1$ will lead to an MLS pair whose PCCF takes on only four values. For such decimations, the upper bounds on the four-valued PCCF is

$$l_4(n) = \frac{d(n)}{2^n - 1}. \tag{4}$$

If the degree of an MLS is not a multiple of 4, then an appropriate decimation of that MLS will yield pairs of MLSs having relatively small three-valued PCCF.[6,10] For MLSs whose degrees are not a multiple of 4, some of decimation factors are of the form $d = 2^k + 1$ or $d = 2^{2k} - 2^k + 1$, where k is chosen such that $n/\gcd(n,k)$ is odd,[6] with gcd() standing for the greatest common divider, the cross-correlation bounds take on preferred small values expressed by

$$l_3 = \frac{2^{\lfloor (n+2)/2 \rfloor} + 1}{2^n - 1}, \tag{5}$$

where $\lfloor x \rfloor$ denotes the integer part of the real number x. This paper will refer to both the three-valued and four-valued MLS pairs as preferred pairs. Their bound values are of practical significance for the SMASM techniques.

TABLE I. Cross-correlation bound values of preferred and reciprocal MLS pairs, of Gold and Kasami sequences.

Degree	Period length	Reciprocal MLS pairs	Preferred MLS pairs & Gold sequences	Kasami sequences
8	255	0.1216	0.1216	0.067
9	511	0.0881	0.0646	
10	1023	0.0616	0.0635	0.0322
11	2 047	0.0437	0.0317	
12	4 095	0.0310	0.0310	0.0159
13	8 191	2.20*E*−2	1.57*E*−2	
14	16 383	1.56*E*−2	1.57*E*−2	7.87*E*−3
15	32 767	1.10*E*−2	7.84*E*−3	
16	65 535	7.80*E*−3	7.83*E*−3	3.92*E*−3
17	131 071	5.52*E*−3	3.91*E*−3	
18	262 143	3.90*E*−3	3.91*E*−3	1.96*E*−3
19	524 287	2.76*E*−3	1.95*E*−3	
20	1048 575	1.95*E*−3	1.95*E*−3	9.77*E*−4
21	2 097 151	1.38*E*−3	9.77*E*−4	
22	4 194 303	9.76*E*−4	9.77*E*−4	4.89*E*−4
23	8 388 607	6.90*E*−4	4.88*E*−4	
24	16 777 215	4.88*E*−4	4.88*E*−4	2.44*E*−4

In addition to the preferred MLS pairs, reciprocal pairs of every binary MLS of degree n also possess small-valued PCCF (Refs. 1, 11) bounded by

$$l_r = \frac{2^{(n+2)/2} - 1}{2^n - 1}, \tag{6}$$

but not limited to three or four values.

Table I lists all bound values of preferred and reciprocal MLS pairs between degree 8 and 24 for an easy comparison. Figure 1 also shows the smallest bounds (logarithmic peak values) of the PCCF of preferred and reciprocal MLS pairs between degree 8 and 24. A careful comparison between columns in Table I and between Eq. (5) and Eq. (6) reveals that the small-valued PCCFs of the preferred MLS pairs are close to that of reciprocal ones for even-numbered degrees, while for the odd-numbered degrees the preferred MLSs are ap-

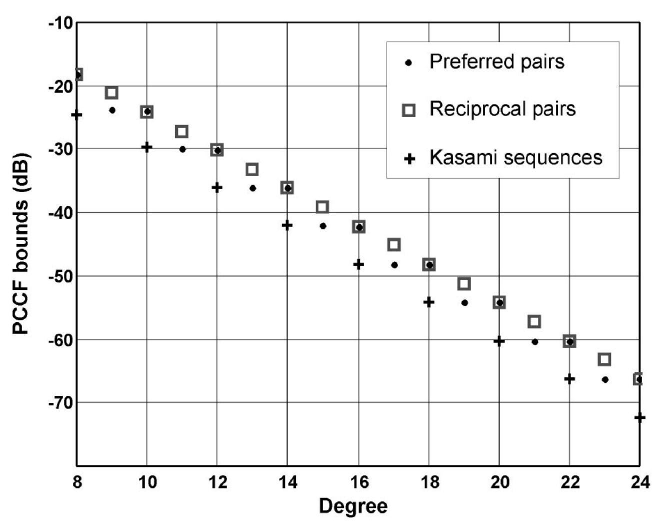

FIG. 1. Bound values of the cross-correlation functions of preferred and reciprocal MLS pairs along with bound values of Kasami sequences. The bound values are expressed in dB relative to the peak value of their auto-correlation function.

proximately one-half that for the reciprocal MLSs. For the current applications, one should not necessarily insist on having three- or four-valued PCCF as long as both their PACF and PCCF approximate the desired condition expressed in Eq. (2).

When specific applications require up to four simultaneous source signals, however, even one preferred MLS pair along with their individual reciprocal pairs will not yield the preferred smallest bound values among four of them. To be more precise, let $\{a(i)\}$ and $\{b(i)\}$ denote a preferred MLS pair of degree 14, and $\{a(i)\}$ and $\{a_r(i)\}$, $\{b(i)\}$, and $\{b_r(i)\}$ denote reciprocal pairs of each individual MLS, respectively. The PCCFs between $\{a(i)\}$ and $\{b(i)\}$, and between $\{a_r(i)\}$ and $\{b_r(i)\}$ are bounded by a preferred value of 0.0157, while the PCCFs between $\{a(i)\}$ and $\{a_r(i)\}$, and between $\{b(i)\}$ and $\{b_r(i)\}$ are bounded at a preferred value of 0.0156, as listed in Table I. However, the PCCFs between $\{a(i)\}$ and $\{b_r(i)\}$, and between $\{b(i)\}$ and $\{a_r(i)\}$ are bounded by 0.03, which is roughly twice the preferred small values.

Practical applications often need more than two simultaneous source channels. While a pair of MLSs selected from two different preferred pairs of the same degree may not possess the preferred small-valued PCCF as listed in Table I and plotted in Fig. 1, some pairs selected from different pairs do. A set of the MLSs for which each pair of the set has this preferred small PCCF values is referred to as a connect set. The largest possible connected set is termed a maximum connect set (MCS) (Ref. 10) and the size of such a MCS is denoted by M_n. In the literature,[6] one can find the values of M_n for $n \leqslant 16$ with $M_n \leqslant 4$. The length of MLSs of these degrees and their reasonably low PCCF bound values are of practical interest in a wide variety of acoustic measurements. However, the number of MLSs in the MCS are strongly limited. One needs to use properly combined MLSs to form Gold and Kasami sequences.

B. Combination of MLS pairs

A binary Gold sequence $\{G_\rho(i)\}$ can be generated by combining a preferred, or a reciprocal binary MLS pair $[\{a(i)\},\{b(i)\}]$ as[10] $G_\tau(i) = a(i) \oplus b(i+\tau)$, with $\oplus$ denoting addition module 2. In stepping τ point-by-point, a large number of Gold sequences result including the preferred MLS pair to yield a finite set $\{\{a\},\{b\},\{G_0\},\{G_1\},\ldots\}$ containing exactly $L+2$ sequences. Sequences in this finite set approximately fulfill the condition expressed in Eq. (2). PACFs of bipolar Gold sequences associated with $\{G_\tau\}$ are peaked at zero lag, but they also possess small-valued sidelobes. The PCCFs of Gold sequences possess the same bound value as that of the sidelobes of the corresponding PACF. In fact, the bound values are the same as those of the PCCF of the preferred and reciprocal MLS pairs from which the Gold sequences are derived.

For even-numbered degree n, a decimation from an MLS $\{a(i)\}$ with factor $\lambda(n) = 2^{n/2}+1$ can lead to a pair $\{a(i)\}$ and $\{c(i)\}$ with $c(i) = a(\lambda i)$. In this case, $\{c(i)\}$ is not an MLS of degree n. A binary Kasami sequence $K_\tau(i)$ can be generated by combining $\{a(i)\}$ and $\{c(i)\}$ as $K_\tau(i) = a(i) \oplus c(i+\tau)$. In similar fashion, a set of Kasami sequences $\{\{a\},\{K_0\},\{K_1\},\ldots\}$ can be constructed[12] containing $2^{n/2}$ sequences. The sidelobes of PACFs and the amplitude of PCCFs among these sequences present even lower bounds, approximately half of those of preferred pairs and Gold sequences

$$l_k(n) = \frac{\lambda(n)}{2^n - 1}, \tag{7}$$

where n must be a positive even number. Figure 1 also shows the bounds values of Kasami sequences in logarithmic scale along with those of preferred and reciprocal MLS pairs for comparison. The power spectral density functions of Gold and Kasami sequences are the same as that of MLSs, being of broadband nature and covering the entire frequency range. In order to demonstrate the excellent correlation properties of Gold and Kasami sequences, Fig. 2 shows the PACF and PCCF of a pair of Kasami sequences, derived from an MLS of degree 14 decimated by a factor of 129. In the example, the PCCF values and the sidelobes of the PACF are bounded by 0.007 87, or 42.1 dB below the peak value of the individual PACF. Unlike MLSs, the PACFs of Gold and Kasami sequences are pulse-like functions with small-valued sidelobes. They approximate the condition expressed in Eq. (2) to the required degree of accuracy, and they are equivalent to the MLSs drawn from MCSs with regard to their usefulness in the SMASM technique.

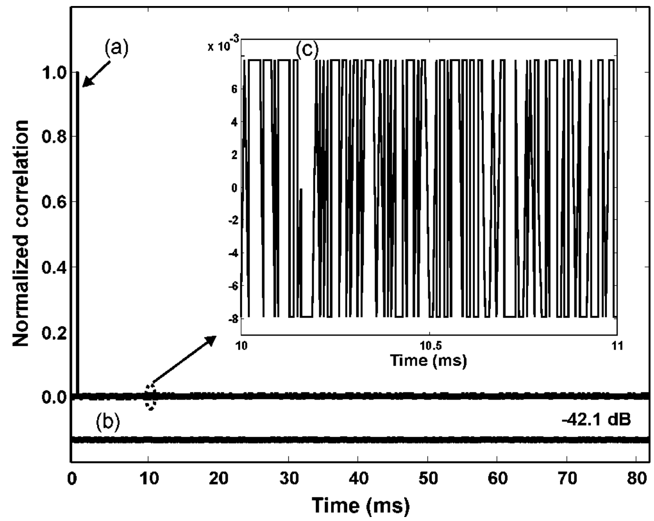

FIG. 2. Normalized correlation functions of Kasami sequences of degree 14. (a) Periodic autocorrelation function (PACF) of one sequence. (b) Periodic cross-correlation function (PCCF) between two Kasami sequences (shifted downwards beneath the autocorrelation function for a convenient comparison). The peak value of the PCCF is 42.1 dB lower than that of PACF. (c) Zoomed presentation of a segment from (a) showing three-valued sidelobes of the PACF. Their peak values are the same as those of the PCCF in (b).

III. SIMULATION AND EXPERIMENTAL RESULTS

In this section we present simulation and experimental results to demonstrate usefulness of the lack of correlation among the MLSs from MCS and combined MLSs. Unlike applications in the spread spectrum technology, where MLS-related sequences are primarily used for coding or modulation, the acoustic measurement technique discussed in this paper employs the sequences directly for acoustic excita-

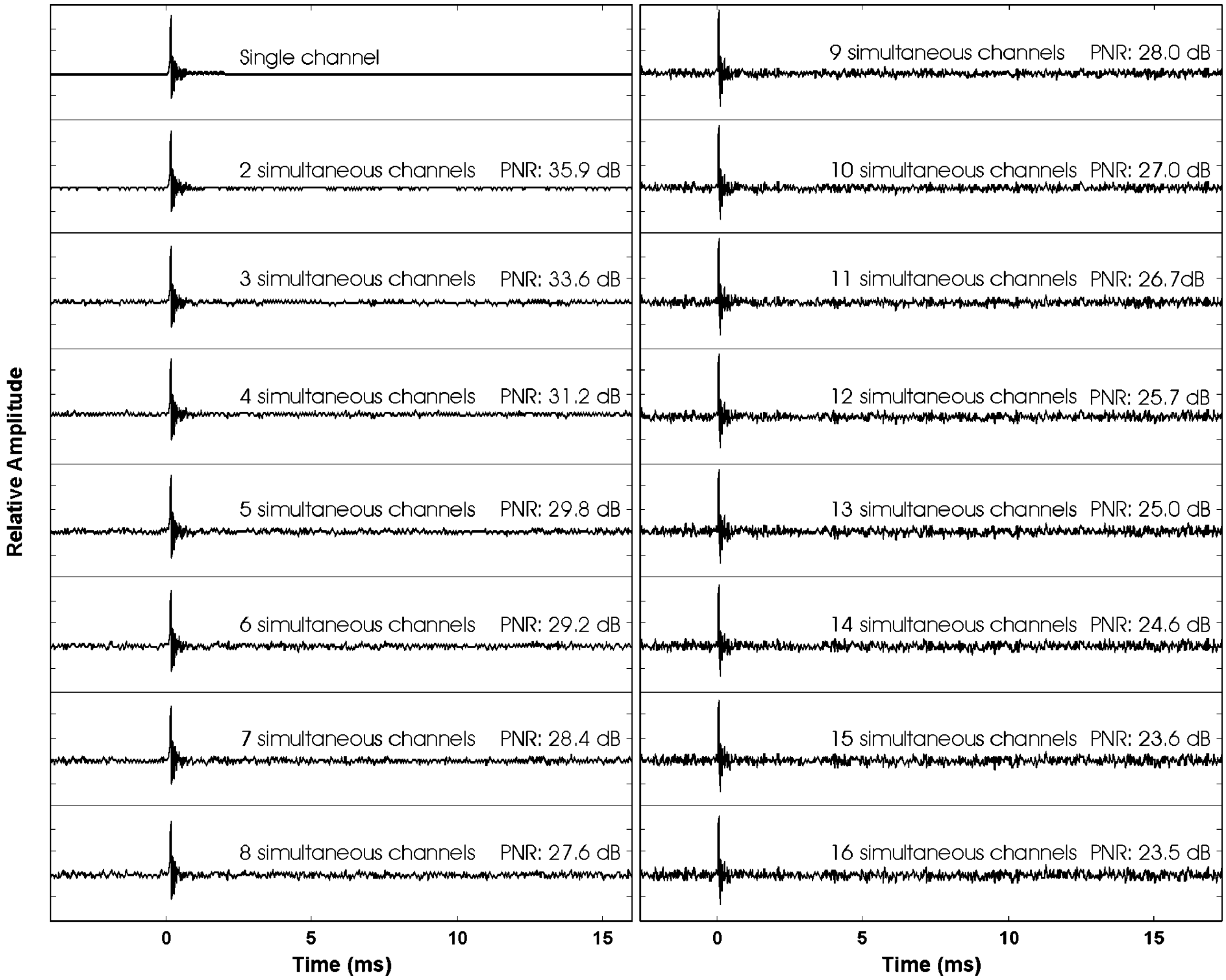

FIG. 3. Simulation results using digital filters to study the noise behavior of simultaneous multiple source measurements, Single channel, 2, 3,..., and 16 simultaneous source channels are simulated. Impulse responses obtained using increasing number of simultaneous Kasami sequences of degree 14 are label by the number of simultaneous source channels and the peak-to-noise ratio (PNR).

tions. The acoustic channel responses are then cross correlated with the excitation sequences. Since these sequences are of length $2^n - 1$, conventional FFT cannot be employed directly. Fortunately the fast MLS transform (based on fast Hadamard transform) can be employed for MLSs and reciprocal MLSs.[1] When using combined sequences, however, one needs to exploit a specialized algorithm[13] for calculating the PCCF.

A. Digital simulation

In order to understand the signal-to-noise ratio achievable in the SMASM applications, we apply a set of 16 bipolar Kasami sequences $\{\{k_1\},\{k_2\},\ldots,\{k_{16}\}\}$ transmitted at a rate of 50 kHz to simulate acoustic channel experiments. Among them, $\{k_1\}$ is a single MLS of degree 14 and $\{k_2\},\ldots,\{k_{16}\}$ are Kasami sequences constructed by combining $\{k_1\}$ and $\{b\}$, with $b(i) = k_1(129i)$. The PCCFs among the 16 sequences possess the same small values as given in Table I.

Eighth-order low-pass Chebyshev filters, each having a cutoff frequency of 16 kHz, are used to simulate individual acoustic channels. Each individual sequence is filtered by one low-pass filter separately. First one, then two,..., then all 16 of the filter responses are summed to simulate the signal that would be received at one point and at a common point from two, three,..., and all 16 simultaneous sources. The summed signals are then cross correlated with the MLS $\{k_1\}$ to obtain the impulse response associated with the source sending $\{k_1\}$. Figure 3 shows the impulse responses of a single source channel and of simultaneous multiple channels.

To quantify the quality of the impulse response, we define peak-to-noise ratio (PNR) as the ratio of its peak value to the rms value of noise, which is the difference between the exact value of the impulse response and the value calculated using our technique at each time epoch. More precisely, the signal received at receiver i is

$$r_i = \sum_{j=1}^{16} h_{ij} * s_j . \tag{8}$$

If we now correlate the received signal with s_k, we obtain the following approximation for h_{ik}:

$$\hat{h}_{ik} = h_{ik} + h_{ik} * (s_k \otimes s_k - \delta) + \sum_{j=1, j\neq k}^{16} h_{ij} * (s_j \otimes s_k). \tag{9}$$

Thus, the sequence

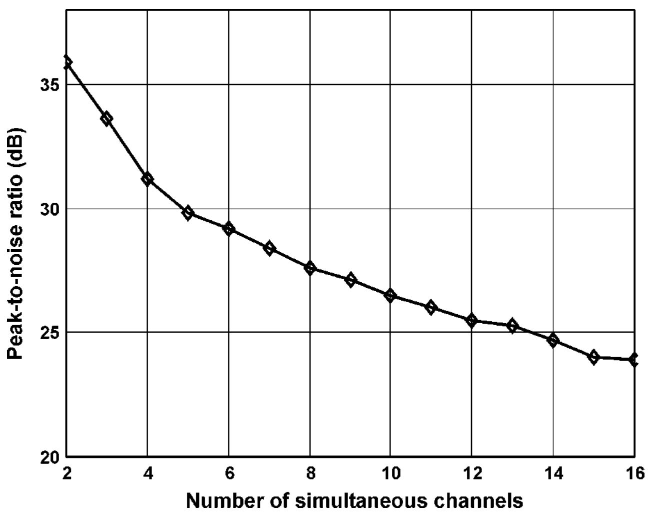

FIG. 4. Peak-to-noise ratios (PNR) as a function of simultaneous source channels, achieved using digital filters to simulate the acoustic channels. PNR obtained using Kasami sequences of degree 14 ranges from 35.9 dB down to 23.5 dB when the number of simultaneous source changes from 2 to 16.

$$e_{ik}=\hat{h}_{ik}-h_{ik}=h_{ik}*(s_k\otimes s_k-\delta)+\sum_{j=1,j\neq k}^{16} h_{ij}*(s_j\otimes s_k), \tag{10}$$

represents the error between the approximation for h_{ik} and its actual value, that is, it is a noise sequence. Thus

$$\overline{E_{ik}^2}=\frac{1}{L}\sum_{z=0}^{L-1} e_{ik}^2(x), \tag{11}$$

is the mean-squared error between h_{ik} and its approximation, $\hat{h}_{ik}$. If the autocorrelation function of s_k were a δ function, and s_j and s_k were uncorrelated for $j\neq k$, then the mean-squared error would be zero. Otherwise, the ratio of the peak value of h_{ik} to the rms noise value gives a measure of the quality of the estimate of h_{ik}.

Figure 4 shows the PNR achieved at individual numbers of simultaneous sources. The simulation results as shown in Fig. 3 and Fig. 4 indicate that the PNR in the SMASM decreases with increasing number of simultaneous source signals. The noise stems from the sidelobes of the autocorrelation and nonzero residuals of the cross-correlation operations as shown in Fig. 2. The higher the degree of the sequences used, the higher the PNR will be, as Fig. 1 implies. In a similar fashion, when cross correlating the resulting receiver signal in the simultaneous multiple source mode with MLS $\{k_2\},\{k_3\},\ldots$, the impulse responses $\{h_{12}\},\{h_{13}\},\ldots$ would be resolved.

The fast M-sequence transform[1] is used in an efficient calculation of cross correlation between MLS $\{k_1\}$ and the summed filter responses. When resolving impulse responses associated with combined MLSs $\{k_2\},\{k_3\},\ldots$, the fast MLS transform cannot be used, but a specialized algorithm may be used for calculating PCCF.[13]

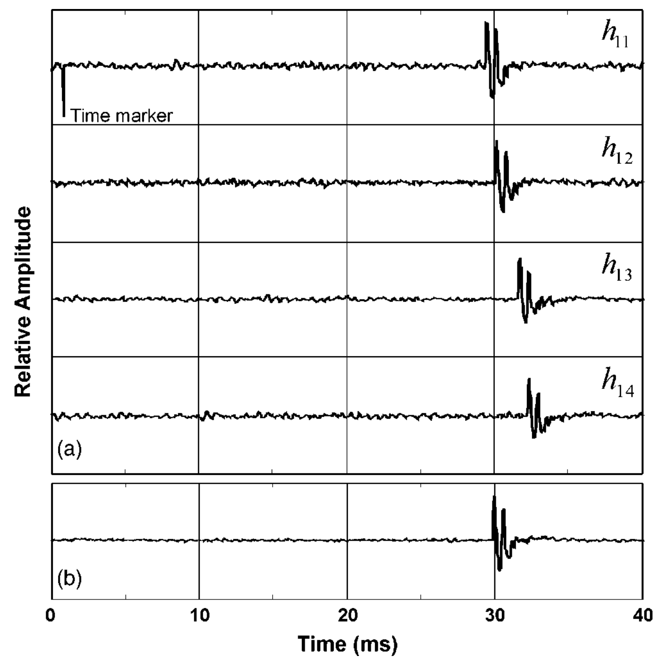

FIG. 5. Experimental results of times of flight for outdoor sound propagation analysis. (a) Four simultaneous sound sources and a single receiver. (b) Impulse response between a single source (#2) and the receiver (for comparison with h_{12}).

B. Experimental results

We now present a group of field measurement results achieved from exploratory experiments conducted outdoors. Four loudspeakers, which serve as sources, are separated 2 m from each other along a straight line. Perpendicular to this line, a microphone (receiver) is set 10 m away from the first sound source. The receiver to the farthest source is about 11 m. The sound sources and receivers are set 1.1 m above the outdoor ground surface. The sampling frequency for these measurements is 25 kHz. Four MLSs' selected from the MCS of degree 13 are used to simultaneously drive the sound sources. Figure 5(a) illustrates impulse response segments of the first 40 ms associated with four simultaneous sound sources. They are obtained by using the fast MLS transform[1] of the received receiver signal with each of the individual MLSs'. These simultaneous measurements of channel impulse responses yield slightly different arrival times due to different propagation distances of the acoustic wave, corresponding well to the geometry of the setup. Figure 5(b) shows the measured single source impulse responses of the channel between the second source and the receiver when only source two is energized. Although the single source measurement yields a better PNR, the single- and multiple source measurements clearly yield similar results.

An achievable PNR depends upon, among other things, available source power, the sensitivity and bandwidth of the sensors or receivers, the attenuation of the propagation channels, and the length of sequences. In practice, the achievable PNR can also be influenced by other unwanted noise, background noise, nonlinear or time-variant components within the system under test, resulting in a lower PNR. While SMASM cannot provide results of the identical quality that may be obtained through sequential measurement, a welldesigned SMASM can produce results meeting the requirements of all but the most demanding applications.

IV. CONCLUSIONS

The cross-correlation properties of maximum-length sequences (MLSs) and related signals make them good candidates for simultaneous multiple acoustic source measurements (SMASM). In this paper we have demonstrated that, by exploiting the cross-correlation properties of MLSs and MLS-related sequences—Gold and Kasami, in particular—SMASM becomes feasible. Both simulation and experimental results achieved from exploratory outdoor measurement have demonstrated the feasibility of using MLSs and combined MLSs for the SMASM technique. This proposed measurement technique would be especially useful when an application demands simultaneous, rather than sequential, excitation of multiple sources.[3,4] When a sufficient number of single MLSs from a maximum connect set (MCS) can meet the need, a fast algorithm, the so-called fast MLS transform[14–17] can be applied directly to retrieving impulse responses between simultaneous multiple sources and one or several receivers. Otherwise, additional Gold and Kasami sequences can be selected to meet the need. In the latter case, a specialized algorithm, such as the one documented in Ref. 13, must be applied.

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