A modified diffusion equation for room-acoustic predication (L)

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This letter presents a modified diffusion model using an Eyring absorption coefficient to predict the reverberation time and sound pressure distributions in enclosures. While the original diffusion model [Ollendorff, Acustica **21**, 236–245 (1969); J. Picaut *et al.*, Acustica **83**, 614–621 (1997); Valeau *et al.*, J. Acoust. Soc. Am. **119**, 1504–1513 (2006)] usually has good performance for low absorption, the modified diffusion model yields more satisfactory results for both low and high absorption. Comparisons among the modified model, the original model, a geometrical-acoustics model, and several well-established theories in terms of reverberation times and sound pressure level distributions, indicate significantly improved prediction accuracy by the modification. © *2007 Acoustical Society of America.* [DOI: 10.1121/1.2727331]

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I. INTRODUCTION

Recently, a diffusion equation has drawn attention in room-acoustic predictions. Ollendorff[1] first proposed the diffusion equation to describe diffuse sound fields in enclosures. More recently, Picaut[2] and his co-workers[3–5] have extended the application of the diffusion equation model based on the concept of acoustic particles[6,7] to a variety of space types, including elongated space, such as street canyons,[3] single-space enclosures,[4] and coupled-volume spaces.[5] The previous work, however, shows that the diffusion model is only suitable for low absorption coefficients.[3,4] The subject of this work is to present a modification to the diffusion model, as already applied in room acoustics,[3–5] using an improved absorption coefficient from Eyring's equation,[8] which suggests a better prediction of the sound fields in enclosures with both low and high absorption coefficients.

Predicted reverberation times and sound pressure levels (SPLs) are compared among the modified diffusion model, the original diffusion model, and other classical theories for cubic rooms with both uniformly and nonuniformly distributed absorbing surfaces of varied absorption coefficients. For flat and long rooms, a geometrical-acoustics model is used for comparisons of the SPL distributions. The results show that the modified diffusion model improves the room acoustic prediction. Experimental work of SPL distributions for various room types to verify the modified diffusion model is being undertaken.

In Sec. II, the modified diffusion equations are formulated based on the original diffusion equations. Section III briefly discusses simulation results for different kinds of spaces in terms of comparisons among the modified, the original diffusion model, and other approaches. Section IV concludes the paper.

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II. DIFFUSION EQUATION MODIFICATION

The energy balance for a room of volume V containing a sound source of power output $F(\mathbf{r},t)$ can be written by a diffusion equation,[2–4]

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + \frac{cS\bar{\alpha}}{4V} w(\mathbf{r},t) = F(\mathbf{r},t), \tag{1}$$

where $F(r,t)$ is an acoustic source term and D is termed diffusion coefficient for introducing a term of Laplace operator ∇^2 on sound energy density $w(\mathbf{r},t)$ for a nonuniform distribution of the sound energy.[1,2] $\bar{\alpha}$ is the average absorption coefficient of the room, and S is the surface area of the room.

The original diffusion equation is only valid when the absorption is very weak since it is the first-order approximation of an integral equation.[9] In fact, the absorption term $(cS\bar{\alpha}/4V)$ in Eq. (1) is the probability rate of a particle to be absorbed during 1 s when the average absorption coefficient is small.[2,8] However, in room acoustics, the absorption coefficient can be very large, for instance, the audience in a concert hall. In the statistical theory of room acoustics, a similar case is that Sabine equation is usually only valid for low absorption because it essentially represents the first-order approximation of Eyring equation. The later is valid for both low and high absorptions. Reducing Eq. (1) to the classical statistical model[4] by dropping the term with the diffusion coefficient D, a sound energy decay equation is written as

$$\frac{\partial w(t)}{\partial t} + \frac{c\bar{\alpha}S}{4V} w(t) = 0, \tag{2}$$

the solution of this equation is

$$w(t) = w_0 e^{-(c\bar{\alpha}S/4V)t}, \tag{3}$$

where w_0 is the initial sound energy density of the sound source. The solution leads to the well-know Sabine's formula. Changing Eq. (2) to

$$\frac{\partial w(t)}{\partial t} - \frac{c \ln(1-\bar{\alpha})S}{4V} w(t) = 0, \tag{4}$$

its solution leads to the Erying's formula.

In similar fashion, a substitution of $\bar{\alpha}$ by $-\ln(1-\bar{\alpha})$ according to Eyring's formula modifies the original diffusion equation in Eq. (1) to

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) - \frac{\ln(1-\bar{\alpha})cS}{4V} w(\mathbf{r},t) = F(\mathbf{r},t). \tag{5}$$

Solutions of this equation require certain boundary conditions. For rooms with uniformly absorbing surfaces, a homogenous Neumann boundary condition[4]

$$\frac{\partial w}{\partial n} = 0 \quad \text{on } \partial V, \tag{6}$$

can be used with ∂V being the surface of the room. For nonuniformly distributed surface properties, a mixed boundary condition can be derived using an exchange coefficient h.[4] The term h is chosen so that the energy flux through the room boundaries equals the absorption over the whole room due to the absorption term from Eq. (5),

$$\int_V -\frac{\ln(1-\bar{\alpha})cS}{4V} w(\mathbf{r},t) dV = \int_{\partial V} h(S) w(\mathbf{r},t) dS, \tag{7}$$

while the sound field is diffusive, which means the sound energy density is quite uniform in the room, $w(\mathbf{r},t)$ can be treated as a constant and taken out of the integral, thus

$$-\frac{\ln(1-\bar{\alpha})cS}{4} = \int_{\partial V} h(S) dS. \tag{8}$$

For a room with N walls, an h_i can be attributed to each wall S_i with different absorption coefficient α_i in terms of

$$h_i \approx -\frac{\ln(1-\alpha_i)c}{4}, \quad i = 1, \ldots, N, \tag{9}$$

since

$$\ln(1-\bar{\alpha}) = \ln\left(1 - \sum_{i=1}^{N} \frac{\alpha_i S_i}{S}\right) \approx \sum_{i=1}^{N} \ln(1-\alpha_i)\frac{S_i}{S}. \tag{10}$$

The equation to describe the energy exchanges at boundaries is written as

$$-D\frac{\partial w}{\partial n} = h w(\mathbf{r},t) \quad \text{on } \partial V, \tag{11}$$

where $\partial w / \partial n$ is the gradient of $w(\mathbf{r},t)$ along the boundary normal.

Overall, the local diffusion equation and the mixed boundary condition can be written as

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) = F(\mathbf{r},t) \quad \text{in V}, \tag{12}$$

$$D\frac{\partial w(\mathbf{r},t)}{\partial n} - \frac{c \ln(1-\alpha)}{4} w(\mathbf{r},t) = 0 \quad \text{on } \partial V, \tag{13}$$

which is more practical than Eq. (5) along with Eq. (6) because it defines the boundary condition to each specific surface with specific absorption coefficients. Moreover, this boundary condition is also an appropriate approximation when applied to flat rooms or long rooms.[3,4] To solve the diffusion equations, a volume sound source is used.[4]

The difference between the modified diffusion equation and the original form lies in a substitution of the logarithmic absorption term to the absorption term. Section III discusses simulation results indicating that this modified model improves predictions of SPL distributions and sound energy decays in rooms with varied shapes not only for low, but also for high absorption coefficients.

III. SIMULATION RESULTS

This section investigates three basic room shape variations with varying absorption coefficients on interior wall surfaces. For cubic rooms with uniformly distributed absorption properties of wall surfaces, several classic theories are used, since the sound energy density is assumed uniform for totally diffusive sound field. For flat or long rooms, the geometrical-acoustics model is considered because the sound energy density is known to be not uniform.

The original and modified diffusion model are implemented by a finite element modeling software with 1500 mesh elements for cubic rooms and 3000 mesh elements for flat and long rooms. The size of the elements is chosen to be on order of or smaller than one mean free path $4V/S$.[4] Equation (5) along with Eq. (6) and Eq. (12) along with Eq. (13) are solved for different initial conditions,

$$w(\mathbf{r},0) = 0 \quad \text{in } V, \tag{14}$$

$$w(\mathbf{r},0) = w_0 \quad \text{in } V_s \tag{15}$$

in the room under acoustic excitations by the volume sound source to obtain the reverberation time (RT), V_s is the volume occupied by the sound source.

With a time dependent solution $w(\mathbf{r},t)$, the SPL can be expressed as[10]

$$L_p(\mathbf{r},t) = 10 \log\left(\frac{w(\mathbf{r},t)\rho c^2}{P_{\text{ref}}^2}\right), \tag{16}$$

where P_{ref} is equal to 2×10^{-5} Pa. The sound energy decay functions can then be obtained.

To calculate the steady state sound field, Eq. (5) along with Eq. (6) and Eq. (12) along with Eq. (13) are solved for a given sound power W_s of the source, and then $F(\mathbf{r},t)$ is set to be equal to W_s/V_s. With a stationary solution $w(\mathbf{r})$, the total SPL can be expressed as[4]

$$L_p^{\text{tot}}(\mathbf{r}) = 10 \log\{\rho c[W_s/(4\pi r^2) + w(\mathbf{r})c]/P_{\text{ref}}^2\}, \tag{17}$$

where the time variable t is omitted here.

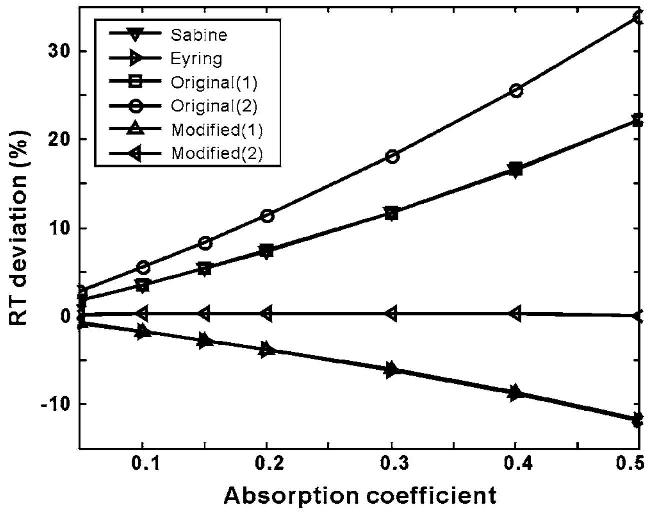

FIG. 1. Deviations of the reverberation times calculated by Kuttruff's formula and other classical methods, along with the original and the modified diffusion models. (1) Homogeneous Neumann boundary condition, and (2) the mixed boundary condition.

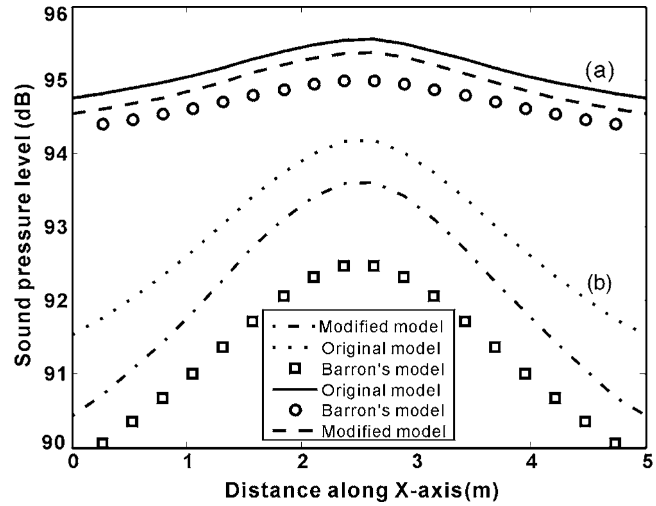

FIG. 2. Comparison of sound pressure level distributions along $y=1$ m ($x=-2.5-2.5$ m, $z=1$ m), by three different models in a cubic room with two configurations of absorption coefficients: 0.1 for each wall (a) and 0.5 for each wall (b).

A. Cubic rooms with uniformly distributed absorption coefficients

A cubic room (-2.5 m$\leqslant x \leqslant 2.5$ m, -2.5 m$\leqslant y \leqslant 2.5$ m, -2.5 m$\leqslant z \leqslant 2.5$ m) is modeled. The source is in the middle of the room with coordinate (0, 0, 0) m. The absorption coefficient is assigned uniformly for all room surfaces. RT is estimated over sound energy level range from -5 to -35 dB.[11] RTs are determined from results of both diffusion models with two kinds of boundary conditions, Sabine's formula, Eyring's formula, and Kuttruff's formula.[8] To compare these different methods in terms of predicted RTs, Fig. 1 illustrates the difference between the results from Kuttruff's formula and other methods. In Fig. 1, the original model with homogeneous Neumann boundary condition agrees more with Sabine's formula, the modified model with this boundary condition agrees more with Eyring's formula. The modified model with mixed boundary condition is very close to the prediction of Kuttruff's formula even when the absorption coefficient is relatively high, and thus has the best performance.

The SPLs are calculated using Eq. (17) along a line having a distance $\sqrt{2}$ m to the source ($y=1$ m, x is from -2.5 to 2.5 m, z is 1 m) by two diffusion models with the mixed boundary condition, and are compared to the results of Barron-Lee's equation which has been verified by the measurements of "reasonably diffusive hall."[12] Figure 2 illustrates comparison results for two different cases. For case (a), the power of the source is 0.01 W and the absorption coefficient of each wall is 0.1. In case (b), the power of the source is 0.02 W and the absorption coefficient of each wall is 0.5. As shown in Fig. 2 the modified diffusion model shows predicted values closer to those determined by Ref. 12.

B. Cubic rooms with nonuniformly distributed absorption coefficients

Again, a cubic room with dimension 5 m$\times$5 m$\times$5 m, but interior surfaces are featured with two different absorption coefficients. The source is in the middle of the room. In this case, only the mixed boundary condition is used because each wall can be assigned to a specific absorption coefficient. Results of RTs are compared to Kuttruff's formula and Kuttruff-Embleton's formula,[13,14] the latter one deals with a room having only two distinct types of surfaces. The results listed in Table I indicate that the modified diffusion model agrees more with Kuttruff-Embleton's formula.

C. Flat rooms

The modified diffusion model and the original diffusion model are compared with a geometrical-acoustics model (CATT Acoustics®). For a flat room with dimension 15 m $\times$15 m$\times$2 m, the source is at (2,5,1) m, the sound power level W_s is 100 dB. The total SPL is obtained using Eq. (17). Figure 3(a) illustrates SPL distributions along the line $x=2.5$ (y is from 0 to 15 m) and at the height 1 m. Figure 3(b) illustrates a sketch of the configuration. The absorption co-

TABLE I. Reverberation times [s] for nonuniformly distributed absorptions in a room predicted by Kuttruff's formula, Kuttruff-Embleton formula, the original diffusion model and the modified one.

Distribution of absorption	Kuttruff	Kuttruff-Embleton	Original	Modified
One wall with 0.9, others with 0	0.8491	0.7148	1.2134	0.6826
One wall with 0.5, others with 0	1.5652	1.4311	1.9232	1.4774
One wall with 0.5, others with 0.1	0.7596	0.7346	0.8716	0.7441
One wall with 0.5, others with 0.2	0.4905	0.4835	0.5723	0.4869

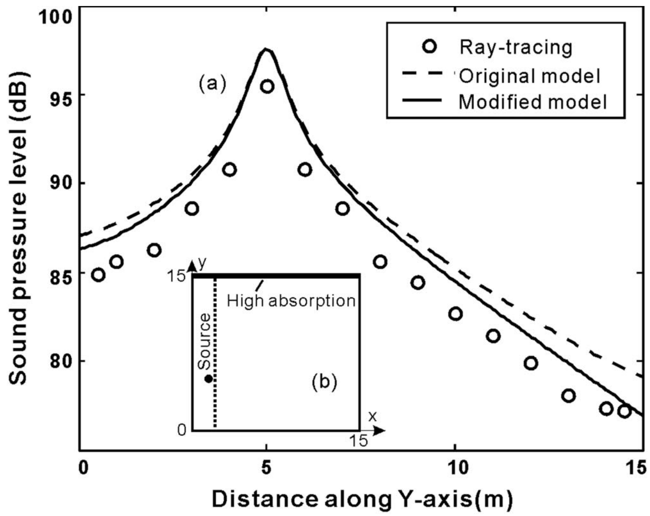

FIG. 3. (a) Comparison of sound pressure level distribution along $x=2.5$ m line in a flat room by the original, modified diffusion model and the geometrical-acoustics (ray tracing) model in a flat room. One wall surface is featured with absorption coefficient 0.8 while the other wall surfaces with absorption coefficient 0.3. (b) Top view of the flat room with dimension 15 m $\times$15 m $\times$2 m, the sound source is at (2,5,1).

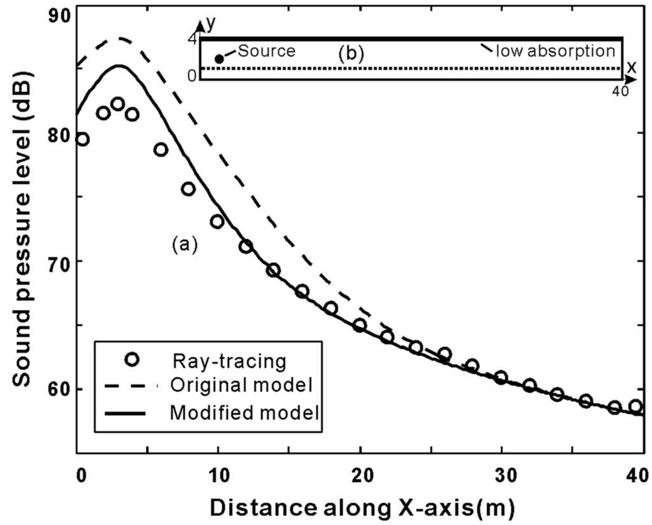

FIG. 4. (a) Comparison of sound pressure level distribution along $y=0.5$ m in a long room by the original, modified diffusion model and the geometrical-acoustics (ray-tracing) model in a long room. One wall surface is featured with absorption coefficient 0.9 while the other wall surfaces with absorption coefficient 0.4. (b) Top view for the long room with dimension 4 m$\times$4 m$\times$40 m, the sound source is at (3,2,3) m, the sound pressure level is calculated along $y=0.5$ m line.

efficient of one wall is 0.8, other walls are 0.3. CATT Acoustics software is used, the number of rays and the ray truncation time are chosen as 5×10^4 and 1000 ms, respectively. These numbers are assumed to be big enough to achieve well-converged results. The reflections are set to be completely diffusive. The modified model agrees better with the geometrical-acoustics model.

D. Long rooms

For a long room with dimension 4 m$\times$4 m$\times$40 m, the source is at (3,2,3) m, the sound power level is 100 dB. Figure 4(b) illustrates a sketch of the room configuration. The SPL distribution is calculated in the same way as the flat room and is plotted in Fig. 4(a) along the line $y=0.5$ m (x is from 0 to 40 m) and at the height 1 m. The absorption coefficient of one wall is 0.4, other walls are 0.9. CATT Acoustics software is again used. The modified model still agrees more with the geometrical-acoustics model.

IV. CONCLUSIONS

This work introduces a modification into a diffusion equation recently applied in room-acoustic predictions, the substitution of Eyring coefficient for Sabine coefficient in the diffusion equation results in more accurate results, especially for high absorption coefficients. Examples of cubic rooms are first given and compared with several well-established classical room-acoustic theories. For uniformly distributed absorption coefficients, the modified diffusion model shows a good agreement with Kuttruff's formula[8] and Barron-Lee's equation,[12] for nonuniformly distributed absorption coefficients, the results of the modified diffusion model are more close to Kuttruff-Embleton's formula.[13,14] At last, the sound pressure distributions of a flat and a long room are simulated. The modified diffusion model yields more similar results to those estimated by the geometrical-acoustics method. Comparisons carried out in this work among the modified model, the original model, the geometrical-acoustics model, and several classical theories indicate that a slight modification of the diffusion equation, as already applied in room acoustics, results in significant improvement in the prediction accuracy.

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