A pseudo-inverse algorithm for simultaneous measurements using multiple acoustical sources (L)[a]

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Simultaneous multiple acoustical sources measurement (SMASM) has been proposed for more effective and reliable identification of acoustical systems under critical conditions [N. Xiang and M. R. Schroeder, J. Acoust. Soc. Am. **113**, 2754–2761 (2003); N. Xiang, J. N. Daigle, and M. Kleiner, J. Acoust. Soc. Am. **117**, 1889–1894 (2005)]. This paper presents a pseudo-inverse algorithm for the SMASM correlation technique as an alternative way of extracting impulse responses of acoustical channels. Simulations and room acoustics experiments are carried out and the results prove the feasibility of the proposed algorithm.
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I. INTRODUCTION

This paper derives a new algorithm for simultaneous acoustic measurements using multiple sound/vibration sources. The simultaneous multiple acoustic sources measurement (SMASM) uses several acoustic sources at the same time to determine impulse responses of linear time invariant systems in the same frequency range. The technique can be used in acoustic delay-time tomography as to investigate sound propagation near the ground surface in outdoor environment.[1,2] Also in room acoustics, room-acoustic parameters are derived from room impulse responses between loudspeakers and microphones. Due to the simultaneous excitation of multiple sound sources and one or multiple sound receivers, the SMASM considers the acoustic system under test as a linear time-invariant multiple-inputs and multiple-outputs (MIMO) system.

In recent years, the maximum length sequences (MLS) technique proposed by Schroeder[3] serves as an effective method in acoustical system identification. Based on MLS's excellent correlation properties, MLS technique is highly immune to extraneous noise and provides a repeatable solution with high accuracy for the single-input and single-output system identification.[4] In the SMASM technique, the MIMO system identification requires several simultaneous excitations associated with multiple acoustic sources within the same frequency range. A previous work[5] presented a technique using reciprocal MLS pairs as excitations in dual-channel measurements using two simultaneous acoustical sources. However, for the complex MIMO system with more sound sources, excitation signal classes with similar correlation properties, but more than two channels have to be adopted to meet the practical requirements of SMASM. The autocorrelation of one coded signal, such as Gold or Kasami sequences,[6–9] is a pulse-like function, while the cross correlation between any two different coded signals in these signal classes is a function with small values relative to the peak value of the autocorrelation function. With respect to this correlation property, a large number of coded signals can be easily derived for the SMASM. Among currently available MLS and MLS-related sequences as applied to a broad range,[8,9] Kasami sequences possess the lowest cross-correlation bound value.[5] This paper uses Kasami sequences as the excitations for the purpose of algorithm verification.

A recent work[10] proposed a specialized fast cross-correlation method for extracting acoustic channel impulse responses. Based on this specialized cross-correlation method, this paper derives a novel algorithm for SMASM using coded signals, such as MLS and MLS-related classes. Different from the specialized fast cross correlation as done in Ref. 10, impulse responses of the MIMO system can be determined using the pseudo-inverse algorithm. The coded signals are widely applied in spread spectrum communications.[9] The pseudo-inverse algorithm has not been documented in major acoustical journals. This paper along with Ref. 10 may piece together a coherent understanding of efficient algorithms for the simultaneous measurements using multiple acoustic sources, which can meet critical needs of some acoustical applications.[1,2]

II. PSEUDO-INVERSE ALGORITHM

A. Simultaneous multiple acoustic sources measurement

Figure 1 illustrates a SMASM scheme, where the vector $\mathbf{s}=(s_1,\ldots,s_n)^T$ stands for the multiple coded signals as system's excitations, and $\mathbf{r}=(r_1,\ldots,r_p)^T$ denotes the system's responses to these excitations, with $()^T$ standing for matrix transpose; n is the number of simultaneous sources, while p is the number of receivers.

With purposely selected excitations, the system identification task is to determine the impulse response matrix $\mathbf{H}$

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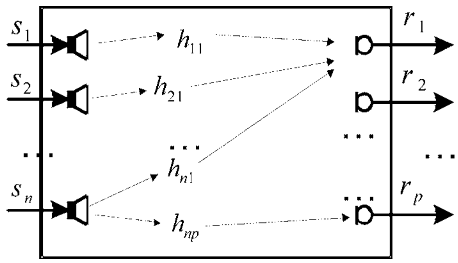

FIG. 1. Simultaneous multiple acoustic sources measurement (SMASM). In SMASM, multiple excitations with acoustical sources $s_1, s_2, \ldots, s_n$ are applied to drive the system under test at the same time; several receivers $r_1, r_2, \ldots, r_p$ are receiving the responses. Acoustical impulse responses h_{ij} are defined between this multiple-input and multiple-output system.

$=[h_{ij}]$ for $1 \le i \le n, 1 \le j \le p$. The MIMO system under simultaneous excitations of multiple sources is represented as

$$\mathbf{r} = \mathbf{H} * \mathbf{s} \tag{1}$$

with $*$ denoting linear convolution.

B. Pseudo-inverse algorithm

The periodic autocorrelation function of binary coded signal is a pulse-like function and their periodic cross-correlation functions (PCCFs) are of small values.[11] A straightforward way is to correlate $\mathbf{s}$ at both sides of Eq. (1). When $\mathbf{s} \otimes \mathbf{s} = \mathbf{\Delta}$, matrix $\mathbf{H}$ can be determined as the cross-correlation between responses and excitations, where $\mathbf{\Delta}$ is a diagonal matrix with a unit sample sequence $\delta(t)$ being each of its diagonal elements, $\otimes$ denotes the periodic cross-correlation.

However, this solution met two major difficulties in practice. One is the direct correlation for SMASM is very time-consuming with n sources and p receivers. A Hadamard transform based MLS transform[4,5] is no more applicable for coded signals such as Gold or Kasami sequences. For this reason, the recent work proposed a specialized correlation algorithm,[10] in order to exploit the fast Fourier transform (FFT) for efficient correlation processing of coded signals of a length being not suitable for FFT. Inspired by the specialized FFT deployment in Ref. 10, this paper derives a pseudo-inverse approach.

The other difficulty lies in the error caused by the assumption of $\mathbf{s} \otimes \mathbf{s} = \mathbf{\Delta}$. Small PCCF bound values of the coded signals,[7,8] imply $\mathbf{s} \otimes \mathbf{s} \approx \mathbf{\Delta}$, introducing inaccuracy into impulse response extraction using the direct cross-correlation method.[7]

The MLS and MLS-related coded signals are of length $2^N - 1$, with N being a positive integer. To exploit the FFT based on the specialized algorithm,[10] original responses and excitations are both appended with $2^N + 1$ zeros, so, s_i and r_j are both augmented to form new sequences s_i', r_j' whose lengths are $L' = 2^{(N+1)}$, being suitable for $\mathrm{FFT}_{(N+1)}$, the subscript $N+1$ of the FFT, and later of the IFFT, explicitly denotes the length of $L' = 2^{(N+1)}$. With the complex signals $\mathbf{R}' = [R_1', \ldots, R_p']$ and $\mathbf{S}' = [S_1', \ldots, S_n']$ in frequency domain after $\mathrm{FFT}_{(N+1)}$, Eq. (1) is expressed as

$$\mathbf{R}' = \mathbf{H}' \cdot \mathbf{S}'. \tag{2}$$

Multiplying $\mathbf{S}'^*$, the conjugated form of $\mathbf{S}'$, on boh sides of Eq. (2) yields

with $\mathbf{\Phi} = \mathbf{S}' \cdot \mathbf{S}'^*$, $\mathbf{\Phi}$ is a n by n by L' matrix. Each element of matrix $\mathbf{\Phi}$ is a vector with L' items. Let $\mathbf{\Phi}^{\#}$ be a pseudo-inverse matrix of $\mathbf{\Phi}$, matrix $\mathbf{H}'$ is approximated by

$$\mathbf{H}' \approx \mathbf{R}' \cdot \mathbf{S}'^* \cdot \mathbf{\Phi}^{\#}. \tag{4}$$

To avoid possible singularities, singular value decomposition (SVD) is used to calculate the pseudo-inverse matrix. Since the excitation signals are known prior to the acoustical measurement, matrix

$$\mathbf{K} = \mathbf{S}'^* \cdot \mathbf{\Phi}^{\#} = \mathbf{S}'^* \cdot (\mathbf{S}' \cdot \mathbf{S}'^*)^{\#} \tag{5}$$

needs to be calculated only once in advance and saved in memory, so that

$$\mathbf{H}' \approx \mathbf{R}' \cdot \mathbf{K}. \tag{6}$$

Applying the inverse FFT of length L' to each element in $\mathbf{H}'$ yields $h_{ij}' = IFFT_{(N+1)}(H_{ij}')$ with $i = 1, \ldots, n$; $j = 1, \ldots, p$.

According to the specialized correlation algorithm proposed in a recent paper,[10] a point-wise addition of the first L points with the last L points of h_{ij}':

$$h_{ij}(k) = h_{ij}'(k) + h_{ij}'(L' - k), \quad 0 \le k \le L \tag{7}$$

yields the impulse response h_{ij} in time domain for the channel between ith source and jth receiver. Impulse responses can be obtained by convoluting the responses of the system under test with the matrix $\mathbf{K}$ using Eq. (6) and Eq. (7). Since matrix $\mathbf{K}$ is prepared in advance and saved in memory, this algorithm is equally efficient as that of the specialized correlation algorism in Ref. 10 in terms of computational loads when a large number of measurements using the same multiple excitation signals is done.

III. DIGITAL SIMULATIONS

In order to verify the feasibility of the pseudo-inverse algorithm in the SMASM application, digital simulations are carried out to mimic practical measurements. Eighth-order low-pass Chebyshev filters with a cut-off frequency of 18 kHz are adopted to simulate the acoustic channels at a sampling frequency of 50 kHz. Each excitation is shifted for a certain, but different delay time to simulate the different distances from acoustical sources to microphones. Each sequence is filtered by the low-pass filter, and finally the filtered signals are summed up as a collected signal for one receiver. In this way, the simulated, multiple responses of SMASM are obtained. The pseudo-inverse algorithm is used to extract the impulse responses in simulation (similar to Ref. 7). Figure 2 illustrates a set of simulation results employing Gold sequences as excitations, by changing the length of Gold sequences and the number of simultaneous excitations, and impulse responses are obtained. The peak to noise ratio (PNR) of impulse responses as defined in Ref. 10 are illustrated in Fig. 2.

In Fig. 2, degree 12, e.g., denotes Gold sequences of length 2^{12}–1. Obviously, the longer the coded signals, the better the PNR of the impulse responses will be. Also, the more simultaneous channels are adopted, the lower the PNR

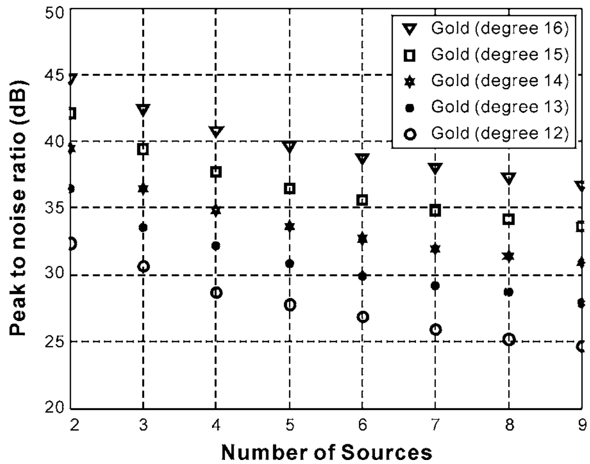

FIG. 2. Peak-to-noise ratio of impulse responses as functions of number of simultaneous sources using Gold sequences of length from $2^{12}-1$ to $2^{16}-1$. Digital low-pass filters are used to simulate acoustical channels.

becomes, due to channel cross-talk noise. Similar simulations are also undertaken with Kasami sequences of different lengths; the PNR of corresponding impulse responses with respect to number of simultaneous excitations are illustrated in Fig. 3. Using Kasami sequences, the SAMSM is able to reach a higher PNR in impulse responses compared with Gold sequences with the same degree.

IV. EXPERIMENTAL RESULTS

The pseudo-inverse algorithm derived above has been used in practical acoustical measurements for verification. Coded signals are directly fed into sources at an update rate of 50 kHz to cover the frequency range of interest. The room acoustical measurement of room impulse responses is carried out in San Patrick Church, Watervliet, New York. In the measurements, four loudspeakers are used as the acoustical sources driven by four Kasami sequences of degree 22, their corresponding matrix **K** in Eq. (6) was prepared in advance. At the receiving end, a binaural artificial-head system with two microphones (left, right ears) is used to capture the

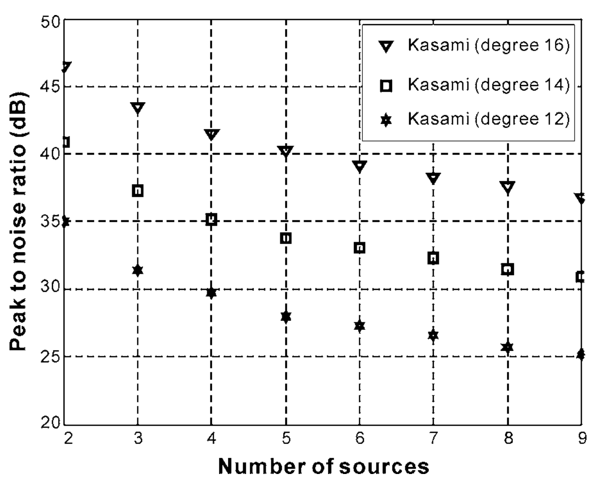

FIG. 3. Peak-to-noise ratio of impulse responses as functions of number of simultaneous sources using Kasami sequences of length $2^{12}-1$, $2^{14}-1$ and $2^{16}-1$. Digital low-pass filters are used to simulate acoustical channels.

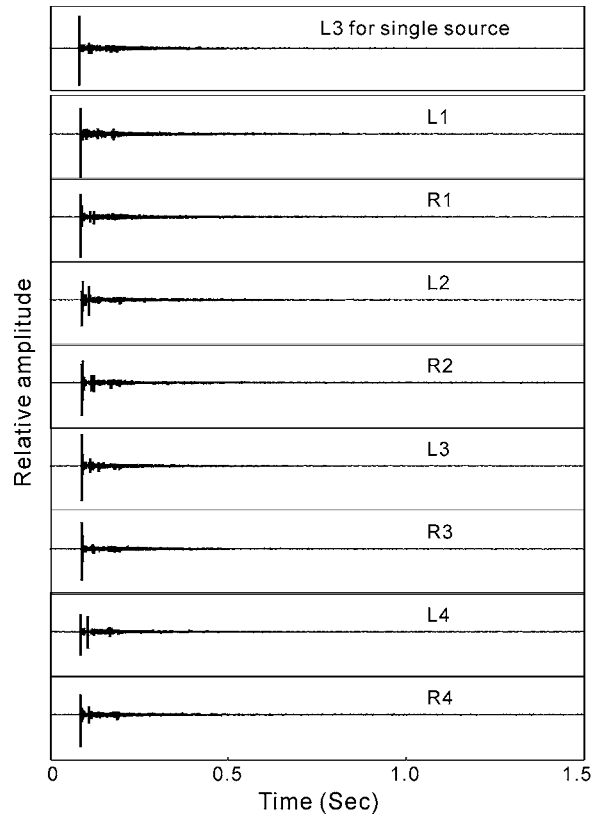

FIG. 4. First 1.5 s of room impulse responses determined by the pseudo-inverse algorithm. Kasami sequences of degree 22 serve as excitations. The figure at the top shows room impulse response of conventional single source measurement corresponding to L3 channel. The other part shows eight room impulse responses of MIMO system with four loudspeakers and two ear microphones; L_1 denotes, e.g., the impulse response between source 1 and the left-ear microphone.

room's steady-state responses to the excitations. The sampling rate of the measurement platform is set to 50 kHz.

After the measurement platform records the whole steady-state responses of the SMASM, the pre-prepared matrix **K** is correlated with the acoustic responses yielding room impulse responses using the pseudo-inverse algorithm [Eqs. (5)–(7)]. Figure 4 illustrates eight room impulse responses (first 1.5 s segments) for channels between the four sound sources and the left and right ear microphones. A single-source impulse response, corresponding to L3, but achieved from one loudspeaker-microphone system is also plotted at the top of Fig. 4 for comparison.

The PNR of impulse responses is primarily determined by the PCCF bounds.[7] It is also influenced by the available source power, the sensitivity and bandwidth of receiver, the attenuation of the propagation channel, etc.[7,11] Nonlinear and time-variant components within the system under test and the background noise will also derogate the measurement accuracy. In this experiment with four simultaneous acoustical sources, the PNR of all room impulse responses can be higher than 45 dB.

V. CONCLUSIONS

This paper proposes an alternative algorithm to extract impulse responses of multiple measurements using coded signals to excite simultaneous sources. Pseudo-inverse algorithm provides a solution for the simultaneous measurement of multiple acoustical channels and implements the extraction of all impulse responses. The proposed algorithm can effectively separate the impulse response of source-receiver pair and characterize this channel even when each single receiver captures the compound responses to several simultaneous sources. The effectiveness of the proposed algorithm is proved by experimental room-acoustical measurements. The pseudo-inverse algorithm proposed in this paper relies on domain transform and the singular value decomposition (SVD); its effectiveness in acoustical measurements for multiple-input and multiple-output systems has been demonstrated. In contrast to the direct cross correlation, it will promise a chance of improving the signal-to-noise ratio using more stable SVD or other numerical techniques inside the current algorithm.

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