Bayesian-based estimation of acoustic surface impedance: Finite difference frequency domain approach

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Acoustic performance for an interior requires an accurate description of the boundary materials’ surface acoustic impedance. Analytical methods may be applied to a small class of test geometries, but inverse numerical methods provide greater flexibility. The parameter estimation problem requires minimizing prediction vice observed acoustic field pressure. The Bayesian-network sampling approach presented here mitigates other methods’ susceptibility to noise inherent to the experiment, model, and numerics. A geometry agnostic method is developed here and its parameter estimation performance is demonstrated for an air-backed micro-perforated panel in an impedance tube. Good agreement is found with predictions from the ISO standard two-microphone, impedance-tube method, and a theoretical model for the material. Data by-products exclusive to a Bayesian approach are analyzed to assess sensitivity of the method to nuisance parameters.

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I. INTRODUCTION

In the architectural acoustics context, two forms of acoustic impedance dominate: the specific acoustic impedance (hereafter characteristic) and the surface acoustic impedance of acoustical boundary treatments (hereafter surface). And while a number of physical mechanisms may be responsible for the determination of surface impedance, including but not limited to density, depth, mass, elasticity, porosity, tortuosity, shear and bulk moduli, structural damping, airflow resistivity, and thermal characteristic lengths, the summary measure of surface impedance describes the interaction of sound with a boundary sufficiently for the purposes of an architectural acoustician, namely, the geometric design and material selection in order to meet acoustic design criteria such as clarity and envelopment.

This surface impedance varies with frequency and the incident direction of the sound. Whereas normal surface impedance is the most prevalent mode investigated and applied in the architectural context, specifying a dampening from normal to grazing incidence that varies according to a sinusoidal angular relation, more general angle-dependent and real-valued, random-incidence models exist. These have been recently deployed in room acoustic parameter prediction through simulation in order to meet design criteria, motivating methods of impedance estimation applicable to general geometries.

This paper details a Bayesian-based approach to the surface impedance parameter estimation problem for which the acoustic model is a finite difference frequency method approximation of the partial differential equations solution of the Helmholtz equation with sound soft, hard, and impedance boundary conditions. While the choice of a numerical approximation to the physics and mathematics of the acoustic problem allows application to general geometries, an experiment-based implementation of the approach is based on the geometry of the impedance tube apparatus for testing normal surface impedance in order to highlight the benefit of the Bayesian-based approach with regard to analytical products which quantify parameter estimate uncertainty and nuisance parameter dependence.

The rest of this paper has the following structure: first the governing differential equations which describe acoustic propagation in the regime of interest are detailed. Boundary conditions particular to the test apparatus of interest are developed to complete the mathematical model; these include sound hard, impedance, and fully absorptive boundaries. A finite difference frequency domain approach casts the continuous problem into a finite difference approximation. The parameter estimation inverse problem is then described within an optimization theory context and a Bayesian network-based search algorithm (modified from Ref. 6) is detailed which provides both a best estimate of the parameter of interest based on experimental data, and an estimate of the distribution of the objective function in order to describe the confidence of the inversion in the best estimate. Experimental results are presented and discussed for two air backed micro-perforated panels in the geometry of the impedance tube. Last, conclusions are drawn.

II. LITERATURE REVIEW

Sound propagation models for architectural acoustics form the basis of virtual prototyping. Whether the model is traditional statistical energy analysis, more spatially accurate geometrical acoustics methods (ray-tracing, image-source), or higher fidelity wave-based acoustics methods [finite-difference-frequency-domain, finite-difference-time-domain, finite element method (FEM) and boundary element method (BEM) models of the wave equation], the gathering of accurate information on the interaction of acoustic waves with
surface materials, typically porous, is essential.7–13 The industry standard practice for determining surface impedance is the transfer function method (TFM).14,15 The TFM utilizes a tube geometry in order to support a one-dimensional free-field propagation assumption; sound in the tube propagates as coherent plane waves. This assumption allows for an analytic solution to the wave equation that is convenient for analysis but limits study to normal incidence. Another drawback is band-limiting: long, wide tubes are required to extract low frequency surface impedance and narrow tubes to extract high frequency surface impedance. Given that actual operating frequency ranges typically diverge from the theoretical frequency ranges, the process of compilation of impedance datasets derived from the TFM is non-trivial.

Methods for evaluating angle of incidence dependence have been developed; however, to date no international standard has been agreed upon. The in situ method for determination of oblique impedance provides an illustration why this might be the case. The method relies upon a spatial Fourier transform, and requires data on two parallel planes close to the impedance surface; it may be applied to partial data, though greater error results from smaller or sparser datasets. The truncation of the domain of dependence results in a reduction the range of incidence angles under investigation. To obtain results of practical use to the designer of acoustic space, the method imposes restrictive requirements: a hemi-anechoic chamber (to supplement the time windowing used to remove parasitic reflections), a directive, dipole source (to minimize the size of sample required), an automated recording positioning system of data acquisition given to record the large number of recording positions (for determination of oblique incidence impedance), and a large (2 m × 2 m) sample size.16 By contrast, the TFM test apparatus is self-contained, has no moving parts and is commercially available.

More recently, methods for impedance estimation in general geometries find a basis in numerical approximations to the sound propagation models, among which the most prevalent are BEM, FEM, and finite difference methods.17–19 These inverse variational methods deduce the parameter of interest from minimizing the residual error between observed field pressure data and predicted pressure given some best guess as to the surface impedances of the boundaries. To date, the acoustic community has employed inverse variational methodologies to solve mostly exterior problems.20–22 For the problem of finding surface impedance of a boundary from field data, lines of research other than the authors’ are being conducted by the United States National Aeronautics and Space Administration (NASA), by the School of Aeronautics and Astronautics at the University of the Bundeswehr Munich (UBM), and by the United States Sandia National Laboratory.23–25 The NASA studies focus on normal impedance in high velocity grazing flow using a quasi-Newton search and a FEM approximation and have produced the results sufficient for their domain, but perhaps too coarse for the architectural acoustics context. The UBM focus is automotive cross-section geometries, using a hybrid evolutionary/quasi-Newton search and an FEM approximation on simulated data.

In general, acoustics inverse numerical method search has been dominated by regularized, classical optimization methods (Newton and quasi-Newton methods) and more modern gradient-based methods applied to the direct or adjoint problems. Such methods work well in the neighborhood of a local maximum. The general numeric inverse acoustic impedance fit function, however, is “peaky,” meaning that many local best fits exist.24 Excellent prior knowledge is therefore required. Bayesian-based methods do not suffer from such a requirement. To date, the most advanced parameter estimation methods, which include such algorithms, namely, Bayesian methods, have not been applied, though they provide an optimization framework that is more robust, more efficient, and more suitable to the gathering of experimental data.26,27 Such Bayesian methods determine the most likely values of the parameters for a model, given a model and data observed in a physical system which is purported to be reasonably well described by that model; more technically, they link inference to probability statistics of observations by defining a joint probability model for all observables (data) and unobservable (parameters) quantities, calculating the probabilities of the unobservables given the data, called marginals, and then evaluating the fit of the model to the data.28 In the context of the problem of interest, this means not only estimating the best value for the specific acoustic impedance from the data given some model, but evaluating the distribution of that parameter and even the fit of the model against competing models. Many surface impedance studies do not investigate how robust the inversion process is to partial or noisy data;16,24,29,30 some explore dependence of convergence and bias on partial data in an ad hoc fashion only;21 without presenting an analysis of goodness of fit away from the optimal solution, these methods limit potential users’ understanding how noisy, real-world data processed in such a manner might lead to spurious, sub-optimal parameter estimation.

In the Bayesian approach, to sample the parameter space in hopes of both finding the global optimum and sufficiently observing the distribution to provide confidence that the global optimum has indeed been found, a number of methods exist: linear sampling, Metropolis Hastings/Markov-chain Monte Carlo sampling, slice sampling, importance sampling, nested sampling, among others.31 The choice of an efficient sampling method typically relies on prior knowledge regarding the fitness distribution, which may be derived theoretically or from observation of the optimization process using more naive and less efficient methods. Importance sampling recommends itself for the problem at hand given the fairly low dimensionality of the parameter space.32 Importance sampling has greater efficacy and efficiency if the prior distribution used either matches the fitness function well and/or the prior distribution has certain analytic properties. The beta distribution has the desired properties and is sufficiently naive to match the as yet unknown fitness function’s prior distribution. The Bayesian optimization algorithm with decision graphs is an existing framework which has all these desired internals with hurdles to use being that the parameters of interest must be cast as variable strings with discrete
III. PROPOSED METHOD

Architectural acoustics inhabits the linear regime where the propagation of sound in air may be described in the frequency domain by the Helmholtz equation

$$[\Delta + k^2] \Phi(\omega, \vec{x}) = 0,$$

(1)

where $k$ is referred to as the wavenumber and $\omega$ is the angular frequency. The time harmonic acoustic wave equation in the presence of forcing $f$ which is a function of angular frequency and location is the inhomogeneous Helmholtz equation for a scalar velocity potential. On a parameterized spatial domain this equation has the form

$$[\Delta + k^2] \Phi(\omega, \vec{x}) + f(\omega, \vec{x}) = 0.$$  

(2)

The domain of interest, the impedance tube, may be described as a parallelepiped with a source located at one end, a surface patch/point on $\vec{x} = [0 \ 0 \ 0]$, and a material under test at the other end, wherever $\vec{x} = [x_{\text{max}} \ 0 \ 0]$. Boundary conditions for the impedance tube are "essential," where the velocity potential is specified, e.g., where a source of known value is applied, "natural," where the gradient of the velocity potential is specified, e.g., sound hard boundaries, and "ghost layer," a set of points for which the matrix equations satisfy that the dot product of the gradient of $\Phi$ at the boundary is zero. The gradient estimate, centered for stability of the operators, requires an estimate of the velocity potential outside of the domain. Since this value is not part of the domain and not part of the solution per se, it is called a ghost layer. Points on the impedance boundary also require a ghost layer, for which the matrix equations satisfy that the dot product of the gradient of $\Phi$ and the surface normal is equal to the ratio of the characteristic impedance of air and the surface impedance, as at the material under test. These latter may be summarized by

$$Z = \frac{i k Z_0 \Phi}{\nabla \cdot \Phi},$$

(3)

where the fluid specific impedance $Z_0 = \rho_0 c$, with $\rho_0$ the fluid static density. This complex quantity is often described in terms of the real and imaginary components

$$Z = R + iX,$$

(4)

where $R$ is the resistance and $X$ the reactance.

The forcing function may be taken as the point source function

$$f(k, r) = A_x e^{-i k r_s} / 4 \pi r_s,$$

(5)

where $A_x$ represents an amplitude and $r_s$ is the distance from the source to the point of interest.

The Helmholtz equation with forcing and associated boundary conditions may be approximated by replacing the continuous spatial domain with a finite mesh of points and approximating the differential operators with finite differences. For example, the gradient of the velocity potential at $\vec{x}[x_j, y_j, z_k]$ on a uniformly meshed rectangular solid with mesh points separated by $\Delta x$ in each axial direction may be approximated by the "centered" difference,

$$\nabla \Phi(\vec{x}) \approx \frac{1}{2 \Delta x} \left[ \Phi(x_{i+1}, y_j, z_k) - \Phi(x_{i-1}, y_j, z_k) \right] \Phi(x_i, y_{j+1}, z_k) - \Phi(x_i, y_{j-1}, z_k) \cdots \right],$$

(6)

which creates a "stencil" about a point of interest and considers the values nearby. The Laplacian may then be approximated as a composition of this difference operator on itself. Replacing the domain and differential operators thus creates a matrix equation for the interior domain that can be described by "centered" difference

$$K \Phi_n = -F_n,$$

(7)

where $K$ is the so-called "stiffness" matrix describing the spatial dependence of nodes to satisfy the domain and boundary conditions, $F$ is the right hand side including both the forcing and essential boundary conditions, and where $n$ indicates the discretization of both the velocity potential field and forcing function. Known solutions (such as at the source end of the impedance tube) may be excluded from the left hand side, but appear in the formulation on the right hand side in rows where the stencil includes them. Points near the boundary will include their stencil points outside of the domain, for which the solution is assumed zero for a sound hard enclosure. Surfaces on natural boundaries require a "ghost layer," a set of points for which the matrix equations satisfy that the gradient of $\Phi$ at the boundary is zero. The practical inverse problem is a projection problem derived from the discrete problem.
The projection problem may be stated as

$$\min_Z \| \Phi_{\text{observed}} - K^{-1}(Z) F \|.$$  \hspace{1cm} (8)

The scope of the inverse problem may be expanded further to try to establish the nature of the source in the absence of anechoic source test data. Assuming a source model, such as a single monopole, the real and imaginary components of the source (termed $F_0$) may be added to the optimization variable space.

The optimization problem with source as nuisance parameter may be stated as

$$\min_{Z, F_0} \| \Phi_{\text{observed}} - K^{-1}(Z) F(F_0) \|.$$  \hspace{1cm} (9)

Interior nodes of the spatial discretization are subject to the differential equations for the fluid domain. Boundary nodes are subject to the boundary conditions. If the node is subject to essential boundary conditions then its solution is known and so brought over to the equation right-hand side in the forcing vector $F$. It should be noted that to satisfy that the solution to the partial differential system be unique, at least one such essential condition must be specified. If essential boundary conditions, velocity potential at some point(s), are not in fact known but also comprise a nuisance parameter(s) (termed $P_0$) and may be added to the optimization space as

$$\min_{Z, F_0, P_0} \| \Phi_{\text{observed}} - K^{-1}(Z) F(F_0, P_0) \|.$$  \hspace{1cm} (10)

Remaining boundary conditions include natural and impedance, as defined above.

Given the need for global search over a peaky cost function, with the desire for quantification of uncertainty both of the surface impedance and those nuisance parameters incorporated into the model, a hybrid genetic algorithm Bayesian-based method is appropriate. The Bayesian-network based “Bayesian optimization algorithm” may be adapted by casting the parameter of interest and nuisance parameters into binary strings via their IEEE bit representations. These strings then form the “genes” for an evolutionary algorithm. Prior knowledge may condition the number of bits of precision as well as the range of the search space to be explored. A full explanation of the algorithm is beyond the scope of this article and so readers are referred to the literature.

The full search algorithm for the solution to the inverse problem may be made explicit.

1. Specifying an initial set of possible impedance values for the material under test and the nuisance parameter source pressures as binary strings.
2. Mapping these strings to pairs of complex numbers.
3. Updating the stiffness matrix impedance, $K(Z)$, forcing and nuisance parameters, $F_0$, $P_0$, in Eq. (10).
4. Solving the forward problem finite difference frequency domain representation for the predicted field pressures at all locations and extracting the data gathering locations’ predictions as per Eq. (7).
5. Calculating the residual vice, the observed field pressures, and selecting the half of the proposed impedance and source pressure values with smallest residual error, as per Eq. (10).
6. Estimating the Bayesian network that best fits the data.
7. Randomly generating new proposal impedance values and source pressures by traversing the Bayesian network.
8. Repeating the above steps until a maximal number of iterations has been computed or the univariate frequencies of all candidate solution bits agree within some tolerance.

The product of such a search consists not only of a best impedance estimate and nuisance parameters estimates, but a record of the fitness of all candidate solutions for each generation, as well as a Bayesian network for each generation. Thus, at the completion of the optimization process, an analysis of the search may be conducted to provide confidence levels regarding well searched and poorly searched sections of the search domain.

IV. EXPERIMENT AND RESULTS

An apparatus was designed to test the proposed method. A rectangular acrylic “tube” with 12 mm thick walls with density 1.1892 g/cm$^3$ was constructed. The impedance tube had inner dimensions of $1.2 \times 0.1 \times 0.1$ m. The top of the tube consisted of two $1.2 \times 48 \times 12$ mm collar strips, creating an open channel down the top center of the tube. Down this channel slid a $1.8 \times 24 \times 12$ mm rectangular piece of acrylic with an observation aperture, the width of this strip equal to that of the channel. Bonded to this moveable aperture was a larger rectangular piece of acrylic the same width as the tube that continued the observation aperture. These two rectangular pieces formed a setting for a single 6.35 mm diameter microphone and the attending connection hardware. The source wall contained a 9.5 mm diameter aperture small enough that the source term in Eq. (5) might be approximated by a single discrete term. A schematic is provided in Fig. 1. Images of the realization of the construction appear in Fig. 2.

Although the test geometry modifies the industry standard impedance tube geometry, many design criteria of the standard geometry which govern the frequency range of analysis are inherited. The length of the modified impedance tube supports analysis down to 140 Hz; however, to match the source tube impedance, a 72 mm diameter source cone was chosen, so that the low end of frequency operation

FIG. 1. Sketch illustrating experimental apparatus-tube length $L = 1.22$ m, cross-section $a = 0.10$ m, distance from source to closest observation point $b = 0.10$ m, distance from material to closest observation point $c = 0.09$ m, and length of observation aperture $d = 1.03$ m. NB: sliding microphone aperture at top.
of the tube was above the theoretical limit imposed by the geometry.

In the industry standard method, the plane wave assumption breaks down as the wavelength approaches twice the tube width. For the geometry specified, the cut off is 1.6 kHz. While the inverse finite element method of impedance estimation does not suffer from this limitation, the upper end of the frequency range of analysis is limited by other factors: in the theoretical limit, temporal sampling resolution, source geometry, and spatial sampling at a resolution. Available signal capture hardware would have allowed analysis to 6.4 kHz, but conservative spatial oversampling requirements impose a theoretical upper bound for the Bayesian-based inverse finite difference method (BIFDM) of 2.7 kHz. The signal under test was a maximal length sequence of 4095 sample root, repeated 100 times in the recording process to minimize the effect of transient noise. This pseudorandom signal is broadband and so supports simultaneous analysis across the frequency range of interest.

In this study, analysis is further restricted by the low absorption of the material under test at high frequencies. The performance of the proposed method was demonstrated for a micro-perforated acrylic surface (see Fig. 3) backed by an air cavity. Their mechanism of absorption is to couple the acoustic impedance of short tubes to the acoustic mass of the air in a cavity behind the perforated plate.

For a perforated absorber, an acrylic plate of 2.2 mm thickness, with 0.6 mm diameter holes in a square lattice separated at the center by a distance of 6.7 mm, so that the perforated area constitutes 2.5% of the total plate area was laser-cut at the fabrication facilities in the RPI School of Architecture. An air gap of 1.15 cm backed the plate.

The micro-perforated panel was tested in the Sonics Research Lab of N.X. at Rensselaer Polytechnic Institute. Two samples were procured, tested using the setup described above, and analyzed according to a modified TFM in which pressure values are converted to transfer function values by a ratio with the pressure value observed at some fixed point, as well as the proposed BIFDM. Each experiment comprised 140 spatial observation points. The experiments were conducted within a temperature range of 21 and 22 °C, and relative humidity of 33% according to a digital thermometer and hygrometer accurate to within 0.5 °C and percentage points.

The sample frequency in this experiment was 50 kHz. For the BIFDM, the data was transformed to the frequency domain using an FFT size of 2048 samples, so that the frequency precision of the experiment was on the order of 25 Hz. These and other choices related to the optimization algorithm processing allowed computation to complete overnight on a modest quad-core workstation.

An overall summary of the performance of the BIFDM relative to the TFM may be evinced from a comparison of best estimates of the BIFDM analysis and TFM analysis. Figures 4 and 5 compare the best estimates of the resistance and reactance (real and imaginary components) of the surface impedance for the micro-perforated material with backing air gap normalized to the characteristic impedance of the medium, respectively. In each figure, a range of values for the TFM method is calculated from 61 independent pairs of data measurement points.

Figures 6 and 7 compare the relative difference between the BIFDM method and TFM mean estimates of resistance and reactance, as well as the range of the relative difference from the mean among the 61 pairs of data measurement points.

The process for the BIFDM which seeks to determine the optimal predicted surface impedance of the material under test given the model and data produces a set of fitnesses (i.e., values for the cost function to be minimized in 10) for each candidate value. Figure 8 depicts the fitness of the set of points in a close grid about the BIFDM determined value near 1100 Hz as an example. Figure 9 provides marginal fitness distributions for the parameters of interest (real and imaginary normalized surface impedance) and the nuisance parameter related to the source point solution (real and imaginary components of the frequency domain pressure) near 1100 Hz. Distributions have different scales, so these have been normalized as a fraction of maximal fitness on the interval [0,1] to facilitate comparison.
At a particular level, a finite difference solution to the direct problem using the optimal parameters provided by the BIFDM was computed and the transfer function values at the observation points relative to the first observation point were interpolated and compared to the data based transfer function real and imaginary components for a chosen frequency in Figs. 10 and 11.

Finally, a finite difference solution to the direct problem using the optimal parameters provided by the BIFDM was computed for a frequency above the range of analysis for the TFM. The full domain solution magnitude is presented in Fig. 12.

V. DISCUSSION

In general, good agreement is found between the BIFDM and TFM surface impedance estimates. Across the

![Fig. 4. Plot of micro-perforated panel with backing air gap resistance normalized to characteristic impedance of medium versus frequency.](image)

![Fig. 5. Plot of micro-perforated panel with backing air gap reactance normalized to characteristic impedance of medium versus frequency.](image)

![Fig. 6. Plot of the absolute percent difference from the transfer function mean resistance.](image)

![Fig. 7. Plot of the absolute percent difference from the transfer function mean reactance.](image)

![Fig. 8. Plot of the normalized fitness distribution of the micro-perforated panel normalized resistance and reactance about the optimal BIFDM solution at 1.1 kHz.](image)
frequency range of analysis, the BIFDM estimates resistance to within 5% of the mean TFM value, well within the range of difference values exhibited by the TFM (see Fig. 6). BIFDM estimates of reactance are within 50%, also well within the range of values exhibited by the TFM (see Fig. 7) except between 1300 and 1600 Hz, where the TFM method ranges are closer to their mean (see Fig. 5). Whereas resistance is responsible for phase independent attenuation through phase cancellation of the incident and reflected waves in the tube. While both the TFM and BIFDM provide larger errors in estimating reactance than resistance, the BIFDM consistently estimates lower reactance than any TFM estimate, especially in the 1300–1600 Hz range.

Differences between BIFDM and TFM estimates in the upper frequency range are to be expected. Whereas in the lower frequency regime the incident field is expected to be planar for both methods, as the wavelength approaches that of the cross sectional diameter of the tube, the incident field, rather than being planar, begins to display spatial curvature. Beyond the TFM frequency range of operation, that curvature becomes more pronounced (see Fig. 12). Because the ensonification is no longer normally incident in this regime, the performance of the micro-perforated panel may not be predicted accurately by a normal mode method such as the TFM. For the mathematical model of the BIFDM, the impedance model is also assumed normal. Thus the parameter estimated is the effective normal impedance rather than an estimate of the observed impedance as a function of the incidence angles, and so is confounded by the test geometry. A potential mitigation strategy is to allow the parameter of interest to be angle dependent, so that the impedance boundary condition Eq. (3) might be replaced by one in which the sinusoidal dependence implicit in the dot product of the gradient of the velocity potential with the normal vector is exchanged for an angle dependent product. This strategy requires restructuring of the finite difference scheme spatial discretization and stencils a non-trivial modification, but one which is available to the BIFDM and not the TFM.

The effect of the incident field curvature is to excite cross-sectional modes. The observation points are positioned in the model exactly along the central Y-axis of the plot, and so, given the central axis symmetry of the forcing function are unaffected by the Y-axis cross-sectional modes. If observation points, and or the source point, are located somewhat off the central Y-axis, variation of pressure amplitudes, and consequently of the real and imaginary components of the transfer functions would result. A potential solution to this problem then is to introduce nuisance parameters for source and receiver locations. By optimizing over position, the effect these have on error could be extracted, and better parameter estimates made. Again, this is an option available to the BIFDM but not the TFM.

The immediate benefit of the BIFDM over the TFM is the production of marginal distributions out of the optimization process. These marginals are the distribution of the fitness about the optimal value of the parameters plotted, holding all other parameters and nuisance parameters constant at their optimal values. The benefits are twofold: the first relates to searching for the optimal impedance estimate, and the second has to do with identifying and quantifying sources of uncertainty. As to the first benefit, the multidimensional parameter estimation problem fitness distribution appears as a nearly bi-variate Gaussian for the parameter of interest with little skew or covariance between the real and imaginary components (see Fig. 8), indicating that the

![Fig. 9. Plot of the marginal distributions of the resistance and reactance, real and imaginary components of the nuisance parameter $P_0$ about the optimal BIFDM solution at 1.1 kHz.](image)

![Fig. 10. One-dimensional plot of the real component of the transfer function at observation points as compared to the finite difference model prediction using the BIFDM optimal solution at 1.1 kHz.](image)
optimization process is in the neighborhood of a solution; however, the marginals of the resistance and reactance with respect to the real and imaginary components of the nuisance parameter $P_0$ produced near 1 kHz are, by observation of the sheared shape, not Gaussian. Non-Bayesian search methods would struggle to efficiently search the nuisance parameter space for $P_0$, $F_0$ to find the optimal impedance estimate if they relied on an assumption of a quadratic shape for the fitness distribution. Once the nature of these marginals is better understood by exploration, more efficient sampling methods may be recommended. With regard to the second benefit, consider the nuisance parameter $P_0$: This unknown parameter, critical to the model, has an effect of impedance estimate and its uncertainty well understood by study of its marginals: if the real component of $P_0$ were smaller, and the imaginary larger, the reactance estimate would be skew larger. $P_0$ could be replaced by prior knowledge, such as a measurement with a quantifiable error distribution, then the optimal impedance estimate might be more accurately confined.

VI. CONCLUSIONS

The authors have presented a Bayesian-based method of acoustic surface impedance estimation for general interior acoustics geometries. A finite difference frequency domain numerical method provided the basic computational engine for the forward problem, and the inverse problem search domain is explored by means of a Bayesian search approach. The method was applied to the geometry of the impedance tube for comparison with the ISO standard transfer function method.\(^5\) The proposed method well estimated the impedance of an air-backed micro-perforated panel, though estimates deviated from the standard method as the frequency of analysis approached the standard method’s upper frequency of operation. Marginal distributions of the nuisance parameters of forcing and the source strength indicate the need for further investigation into the excitation source. Model modifications were proposed to explore nuisance parameters relating to position uncertainty as well as to extend the model to oblique incidence necessary to properly characterize impedance estimates higher upper frequency ranges of operation of the impedance tube geometry.

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