On the subtraction method for \textit{in-situ} reflection and diffusion coefficient measurements


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Abstract: The subtraction method is a technique critical to several important acoustic measurements. It involves subtracting a reference measurement including only direct sound from one with direct sound and a reflection, to isolate the reflection. The process is very sensitive to environmental conditions, such as changes in temperature, air movement, and microphone positioning. These variations cause small time differences between the reference and reflection measurements, which prevent complete subtraction of the direct sound; the residual direct sound then pollutes analysis of the isolated reflection. This work evaluates methods to compensate for differences to achieve minimal interference from the residual direct sound.

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1. Introduction

The subtraction method is an important step in several acoustic measurements that are important to acoustic practitioners in the field. The first is the \textit{in-situ} absorption/reflection coefficient measurement. This method was suggested by Yuzawa\textsuperscript{1} as early as 1975 and further developed by Mommertz\textsuperscript{2}. Nocke\textsuperscript{3} and Garai\textsuperscript{4} also utilized this technique. ISO Standard 13472 (Ref. \textsuperscript{5}) for the measurement of the absorption of roadway surfaces incorporates the subtraction technique and has been applied to other surfaces such as grass and turf.\textsuperscript{6} The measurement involves taking an impulse response in the free field, and another with the microphone close to the surface under test. The free field measurement is subtracted from the test measurement to isolate the reflection, which is then compared to the free field direct sound to deduce the reflection coefficient from the surface. This measurement technique is of high practical significance because it allows testing of installed materials without the need to remove material to a laboratory for reverberation chamber\textsuperscript{7} or impudence tube\textsuperscript{8} measurements.

The second measurement that requires this technique is the measurement of diffusion coefficients.\textsuperscript{9} Diffusion coefficients are measured by placing a semicircular or hemispherical array of microphones around the sample to be measured, a source outside of the microphone array, and taking impulse responses for each microphone. The polar response of the panel is then distilled into a diffusion coefficient using an auto-correlation function as in Eq. (2). In attaining the polar response from an architectural wall panel for many source incidences, it is often necessary to place the source at grazing incidence to the panel with the microphone opposite. Figure 1 (right) illustrates this condition. This results in the direct sound and panel reflection arriving at the microphone in close succession. Since only the reflection is of interest, a second measurement is taken without the panel present to attain a reference that can be subtracted from the measurement to isolate the panel reflection. This measurement is a simple and practical method to measure surfaces such as those described by Olson and Bradley,\textsuperscript{10} Dadiotis \textit{et al.},\textsuperscript{11} and D’Antonio,\textsuperscript{12} among others. Both of these measurements require taking either two measurements with the same microphone or taking one measurement with two different microphones. In either case, there will be slight differences between the measurements. These differences arise from differences in the response of the microphones, no matter how closely matched
they are, temperature changes and air movement between measurements, and variations in the response of the measurement equipment due to temperature or electrical deviations. This work examines methods to compensate for these small changes.

2. Experiment description

This experiment uses impulse responses obtained from diffusion coefficient measurements. A measurement as shown in Fig. 1 (left) without the diffuser panel present yields the reference measurement; one with the panel present provides the reflection measurement. Subtracting the former from the latter eliminates the direct sound and leaves only the reflection. This paper examines a variety of methods to correct for minor time misalignments between the two signals to attain the optimal result. The subtraction results from each method are then used to calculate the diffusion coefficient, and variations are analyzed. Figure 1 illustrates the measured impulse responses and subtraction results.

3. Subtraction methods

The first scheme examined is the direct subtraction method. This method assumes that environmental conditions are constant between measurements and measurements can be repeated exactly. If this assumption is correct, direct subtraction should produce complete elimination of the direct sound and leave only the isolated reflection. If this is not effective, which is often the case, further processing is necessary.

Several steps may be taken to improve the results. All of these steps require comparing different features of the two measurements. To make this comparison more accurate it is desirable to oversample both signals so corrections of less than one sampling point can be made; after subtraction the signals are downsampled to their original sampling frequency.

The simplest correction is to calibrate the signals so that the amplitudes of the direct sound peaks are the same. This ensures that differences in magnitude do not skew the results and do not account for time misalignments. Aligning the signals in time further improves the subtraction results. This can be done in several ways. Utilizing the peaks of the direct sounds as a reference point and aligning them in time can help correct for overall shifts, but a small shift in the peak may not accurately represent the shift of the whole signal. To incorporate a larger portion of the signal into consideration, cross-correlation of a few milliseconds of the signals, centered on the direct sound peaks, may be effective. The location of the peak of the cross-correlation function will indicate how much to shift one signal in relation to the other to attain...
the best alignment of the direct sounds. However, the cross-correlation function is often skewed by the presence of the reflection within the direct sound component. This is often the case when measuring the reflection coefficient of a rigid surface when the microphone is close to the surface.

4. Subtraction optimization

The optimized approach to achieve the best alignment includes all of the previous schemes, then shifts the signals across each other several points to either side of the shift indicated by the cross-correlation function. By subtracting the reference from the reflection measurement at each of these locations, the most effective result can be found. This is performed at a wide range of oversampling rates in order to have the ability to shift the signals by any fraction of a sampling point. So, for an oversampling rate of 2, the signals are shifted four points in either direction; for 3, six points in either direction; etc. This is repeated up to an oversampling rate of 20 or more, so that every fraction of shift within the four points around the peak correlation is investigated. There is some redundancy since, for example, 10/20 is the same as 1/2, but the advantage is that a step of, say, 1/19 of a point is investigated. At each shift the signals are subtracted and the residual direct sound is compared. More thorough organization of the oversampling rates could improve calculation time by removing the redundant steps. For a limited number of oversampling rates and shifts, a gridded search to find the most effective subtraction is sufficient to obtain optimized results; in more detailed investigations a genetic algorithm can be exploited to improve computation time.

5. Measuring subtraction effectiveness

The process above provides many varying subtraction results; in order to choose the best result, a metric of success is required. As the goal of the operation is to remove the direct sound, subtraction effectiveness can be measured by the percent reduction, or decibel level reduction in the direct sound from the measurement to the result. Specifically, the sum of the energy within 0.5 ms of either side of the direct sound can be compared before and after subtraction to find the effective reduction. Equation (1) defines the reduction factor $R$.

$$R = 10 \log_{10} \left( \frac{\sum_{ds-0.5 \text{ ms}}^{ds+0.5 \text{ ms}} (S_{\text{ref}})^2}{\sum_{ds-0.5 \text{ ms}}^{ds+0.5 \text{ ms}} (S_{\text{result}})^2} \right),$$

where $ds$ is the time of the direct sound peak, $S_{\text{ref}}$ is the reference measurement, and $S_{\text{result}}$ is the subtraction result.

Certainly, if direct subtraction provides suitable reduction in the direct sound component, no further processing is necessary. The level of the residual direct sound in relation to the level of the reflection should also be considered. The smaller the reflection component, the greater influence a given residual direct sound will have. This is particularly important in the in-situ measurement of absorptive materials and grazing source incidence diffusion coefficient measurements. In both cases the reflection is small and also within close proximity to the direct sound.

6. Results

Seventy-two sets of measurements were processed with five different subtraction schemes. The schemes, as described above, are direct subtraction, peak amplitude matching (PAM), peak amplitude and time matching (PATM), peak amplitude matching with cross-correlation alignment (PACA), and the optimized method (OM). An example of the results from two different methods is shown in Fig. 2 (left). Calculating the reduction factor $R$ per Eq. (1) for each mea-
measurement and subtracting the direct method reduction factor yield the improvement offered by each method. Table 1 lists the improvement results for the 72 measurement sets processed. Negative values indicate results that are worse than direct subtraction.

Peak amplitude matching provides some improvement, and since this scheme is so simple to implement, it should be used on all subtraction measurements. Note that aligning the peaks of the direct sound provides no additional improvement. This indicates that the exact alignment of the peaks does not correlate to the best time alignment of the entire direct sound. Cross-correlation, used blindly, provides generally worse results than if the signals had been subtracted directly. This is possibly due to the influence of the reflection in the correlation, which could skew the correlation peak due to the mismatch between the reflection and background noise after the direct sound. Finally, the optimization method shows a marked improvement over direct subtraction, sometimes as much as 12 dB. Where the optimization method failed, as indicated by the −1 dB improvement, was on the rare measurement where direct subtraction was extremely accurate; these measurements showed direct method $R$ factors above all others.

7. Effect on the diffusion coefficient

Since the diffusion coefficient uses auto-correlation of many channels, per Eq. (2), it is susceptible to being skewed by one faulty channel.

Table 1. A comparison of the improvement ($R_{\text{method}} - R_{\text{direct}}$) provided by each subtraction scheme: PAM, PATM, PACA, and OM.

<table>
<thead>
<tr>
<th>Method</th>
<th>Minimum (dB)</th>
<th>Mean (dB)</th>
<th>Maximum (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAM</td>
<td>−1.6</td>
<td>0.4</td>
<td>4.5</td>
</tr>
<tr>
<td>PATM</td>
<td>−1.6</td>
<td>0.4</td>
<td>4.5</td>
</tr>
<tr>
<td>PACA</td>
<td>−29.3</td>
<td>−2.5</td>
<td>4.5</td>
</tr>
<tr>
<td>OM</td>
<td>−1</td>
<td>2.6</td>
<td>12.1</td>
</tr>
</tbody>
</table>
\[ D_\Theta = \frac{\left( \sum_{i=1}^{n} 10^{L_i/10} \right)^2 - \sum_{i=1}^{n} (10^{L_i/10})^2}{(n-1) \sum_{i=1}^{n} (10^{L_i/10})^2}, \]  

(2)

where \( i \) is the microphone position, \( L_i \) is the level at \( i \) microphone, \( n \) is number of microphones in the array, and \( \Theta \) is the source incidence.

In a semicircular array of 72 microphones, if the subtraction method fails on one or two channels, a spike can occur in the polar response. This spike is interpreted as highly directional reflection, and the diffusion coefficient will be artificially low. Figure 2 (right) shows a polar response that has a spike from failed subtraction and the corrected response calculated from optimized subtractions. The effect on the diffusion coefficient is pronounced.

Table 2 lists diffusion coefficients of a quadratic residue diffuser designed for 600–1600 Hz with the source at 60° incidence for each subtraction method. The effect of failed subtraction becomes most evident above 400 Hz where all of the non-optimized methods yield erroneous results. Even below this threshold, the optimized method, which eliminates the spikes, shows a higher diffusion coefficient indicating a more uniform polar response. The consistency of the optimized results also indicates that the method is less subject to variations in the quality of the measurement from each channel of the semicircular array.

8. Concluding remarks

The subtraction method is particularly sensitive to variations in environmental conditions between the reference measurement and the reflection measurement. Aligning the signals from these measurements in amplitude and time can eliminate the differences to provide clean subtraction results. This is a critical step in both in-situ reflection measurements and diffusion coefficient measurements; if the subtraction is not effective, the results from both measurements will be skewed if not entirely erroneous. This complication is amplified in diffusion coefficient measurements because the measurement gauges consistency across many channels. Shifting the signals across one another at various oversampling rates to attain the highest reduction factor is an effective method to determine the optimal subtraction result.
References and links