Sound Decay Analysis in Acoustically Coupled Spaces Using a Re-Parameterized Decay Model

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Abstract. The sound energy decay characteristics of coupled spaces are of increasing interest to architectural acousticians. Coupled spaces occur naturally in concert halls and theaters due to the presence, for example, of balconies, orchestra pits, and stage houses. In addition, many new halls have incorporated hard chambers coupled to the primary space to achieve a flexible variation of the acoustics. In certain conditions, sound energy in these coupled spaces decays with two or more distinct exponential rate constants. The presence of multiple decay rates can have a distinct impact on a hall's perceived acoustical quality. In previous papers [1, 2] we describe our initial work to apply Bayesian inferential methods to the problem of determining multiple decay times in coupled spaces using Schroeder energy decay functions derived from measured room impulse responses. In our previous work, relatively little prior information about parameters of the decay model is incorporated in the inference calculations in spite of the fact that much information regarding both the possible range of the parameters and the relationship between the parameters is known. In this paper we describe our recent efforts to incorporate this prior information into the inference calculations by re-parameterizing the multi-exponential decay model.

INTRODUCTION

Acoustics in coupled spaces have long been studied in the context of architectural acoustics. The orchestra pit, the stagehouse, and balconies in an opera house or theater coupled to the main floor are common examples of spaces for performing art. In every-day life, an apartment with a door opened to a hard staircase can cause an unusual auditory experience due to the long reverberation time of the coupled staircase space. In certain conditions, the sound energy in coupled spaces decays with two or more distinct exponential rate constants. This property is used in a growing number of positively received halls that incorporate secondary hard chambers [3]. These hard chambers are partially coupled to the primary space, simultaneously achieving clarity and reverberance, two important, yet conflicting, auditory attributes. The opening and closing of combinations of these secondary chambers has become an important tool for generating the range of acoustic conditions needed for the widely varying music performed today.

In the 1930's Eyring [4] investigated coupled rooms, and observed experimentally that sound energy in two coupled rooms decays at two different rates in certain conditions. Eyring further observed that energy decay functions plotted on a logarithmic scale are not generally linear for coupled spaces having different natural reverberation times or even for a single room with non-uniformly distributed absorption and no diffusing scheme. This means that the sound energy decay in a single-space room can be of multi-rate character. In addition, a single-rate energy decay can also be observed in coupled spaces, depending on the size of the coupling aperture, the position of the sound receiver and the natural reverberation time of each space.

To better understand and control the acoustics in coupled spaces, it is vital to have an effective and efficient technique for determining both the number of decays and the value of the decay times. This is traditionally accomplished, with difficulty, by visual inspection, since no algorithmic approach has been available. Our recent works [1, 2] have applied Bayesian inferential methods, based on Bretthorst's approach [5], to more efficiently determine the decay times in coupled rooms using Schroeder's energy decay functions [6] as data along with the multi-rate decay models established in [1] and [7]. In these works, relatively little prior information about parameters of the decay model is incorporated in the inference calculations in spite of the fact that much information regarding both the possible range of the parameters and the relationship between the parameters is known. After a brief introduction to room impulse responses and Schroeder's decay functions, we describe our recent efforts to incorporate prior information into the inference calculations by re-parameterizing the multi-exponential decay model.

ROOM IMPULSE RESPONSES AND ENERGY DECAY

For the experimental determination of decay parameters such as decay times in coupled rooms, the widely used maximal-length sequence correlation technique [8] provides a room impulse response (RIR) $h(t_i)$ defined between the sound source and the receiver. Here, time is treated as a discrete variable t_i. In the maximal-length sequence correlation technique, use of the fast M-sequence transform yields RIRs with a high signal-to-noise ratio[1]. Schroeder [6] introduced a method for integrating the squared RIR which yields a high precision sound energy decay function:

$$d(t_i) = \frac{1}{N}\sum_{s=t_i}^{t_{max}} h^2(s), \tag{1}$$

where

$$N = \sum_{s=0}^{t_{max}} h^2(s). \tag{2}$$

Fig. 1 (a) illustrates a room impulse response measured in scale-model coupled rooms. Fig.1 (b) illustrates a Schroeder decay function evaluated from Fig.1 (a) using (1). RIRs measured experimentally are invariably contaminated with background noise. Following [7], the background noise is supposed to be additive, with mean-square value $\langle n^2 \rangle$. By

[1] Use of the fast M-sequence transform also facilitates a rapid measurement procedure.

applying Schroeder's backward integration to the RIR written as a noiseless RIR, $h_f(t_i)$, plus independent background noise (see [1, 7] for more detail), the normalized decay function $d(t_i)$ can be expressed as

$$d(t_i) \approx \frac{\langle n^2 \rangle}{N}(t_{max} - t_i) + \frac{1}{N}\sum_{s=t_i}^{t_{max}} h_f^2(s). \tag{3}$$

In the expression above, t_{max} denotes the upper limit of the Schroeder integration due to the finite length of the measured RIR. The second term on the right-hand side of (3) comes from the noise-free RIR and describes the decay process of the sound energy. For coupled rooms, a multi-rate decay function generally results [1, 7, 9]. This can be modeled as:

$$F_k(\mathbf{X}, t_i) = (\langle n^2 \rangle_s / N)(t_{max} - t_i) + \sum_{\ell=1}^{k} x_{2\ell-1} \exp(-t_i / x_{2\ell}). \tag{4}$$

where $\mathbf{X} = \{x_1, x_2, \ldots, x_{2k}\}$ and $x_{2\ell} = T_\ell / 13.8$. Here T_ℓ denotes the ℓ^{th} decay time to be estimated. In (4) the parameter of the background noise term in (3) is assumed known and is calculated by replacing $\langle n^2 \rangle$ with the sample mean of n^2, $\langle n^2 \rangle_s$, determined using the last 20% of the measured RIR.

MULTI-RATE DECAY IN ROOM ACOUSTICS

In the present paper, a set of these decay models $\mathbf{F} = \{F_1, F_2, \ldots, F_M\}$ is under consideration as given in (4). As a specific case of a double-rate Schroeder's decay function, the decay model with $k = 2$, reads:

$$F_2(\mathbf{X}, t_i) = (\langle n^2 \rangle_s / N)(t_{max} - t_i) + x_1 \exp(-t_i / x_2) + x_3 \exp(-t_i / x_4). \tag{5}$$

The first term on the right-hand side of (5) is associated with background noise in experimentally measured room impulse responses from which Schroeder's decay functions are calculated. It results in the characteristic curvature that occurs towards the upper limit of the integration in a logarithmic plot of Schroeder's decay functions. This curvature, illustrated in Fig. 2 (a) and discussed in [1] and [7], can impede the determination of the decay parameters. The decay model in (4) is in the form of a general linear model. It describes sound energy decay in enclosed spaces after a steady-state sound excitation in the spaces is switched off. Therefore, only positive-valued linear parameters are of interest for decay time determination in architectural acoustics practice[2][2].

As observed by Cremer, Müller, and Schultz [9], architectural acousticians are primarily concerned with the conditions:

$$0 < T_1 < T_2 < \cdots < T_k \text{ and } x_1 > x_3 > \cdots x_{2k-1} > 0. \tag{6}$$

[2] There may exist other acoustical situations without this restriction.

Fig. 2 illustrates two contrasting examples simulated using the double-rate decay model in (5). A normalized time scale is used for simplicity. In Fig. 2 (a), the Schroeder decay function fulfills the conditions in (6) with $T_1 = 0.5$, $T_2 = 1.0$ and with $10\log_{10}(x_1) = 0$ dB, $10\log_{10}(x_3) = -6$ dB and $10\log_{10}(\langle n^2\rangle/N) = -45$ dB. In Fig. 2 (b), the decay parameters of the decay function break the conditions in (6) with the same linear parameters as those in Fig. 2 (a) but with $T_1 = 1.0$ and $T_2 = 0.5$. This double-rate decay function is very close to the single-rate one with $T_1 = 0.96$ as plotted in Fig. 2 (b) for comparison.

Often the decay function in Fig. 2 (b) represents a sound decay process in coupled spaces sensed by a sound receiver in the primary space when the secondary space possesses a shorter natural reverberation time than that of the primary space. In this situation, the coupling aperture acts as an absorption area resulting in a shorter decay time than the natural reverberation time in the primary space. It is therefore reasonable to consider the energy decay in the primary room as a single-rate decay when evaluating the room's acoustical quality.

BAYESIAN FORMULATION AND RESULTS

Previous works [1, 2] apply Bayesian inferential methods based on Bretthorst's approach [5] to architectural-acoustic applications. The prior information expressed in (6), however, was not incorporated into the Bayesian approach in these works. In this present work, we take this prior information into account by re-parameterizing the Schroeder decay model:

$$F_k(\mathbf{Y},t_i) = \frac{\langle n^2\rangle_s}{N}(t_{max}-t_i) + y_1\exp(-t_i/y_2) + \sum_{\ell=2}^{k}\frac{y_1}{\prod_{j=1}^{\ell-1} y_{2j+1}}\exp\left(\frac{-t_i}{\sum_{j=1}^{\ell} y_{2j}}\right). \quad (7)$$

In this re-parameterized model, the coefficients of exponential decays will decrease with increasing decay terms when $y_{2j+1} > 1$ while the decay times are increasing when $y_{2j} > 0$ for $j \geq 1$. As a result, this model incorporates the prior information and relationship between decay parameters into the Schroeder decay model and combines (4) and (6) into a single parametric model with only the constrains $y_1 > 0$, $y_{2j} > 0$, and $y_{2j+1} > 1$ for $j \geq 1$.

In Bretthorst's method for calculating the posterior probability for the models, the model parameters are assigned a Gaussian prior that is broad compared to the likelihood. For convenience, the substitution

$$y_\ell = \frac{\min\{y_\ell\} + \max\{y_\ell\}\exp(z_\ell)}{1+\exp(z_\ell)} \quad (8)$$

is made in (7) yielding $F_k(\mathbf{Z},t_i)$ with each z_ℓ being dimensionless and of the same scale. This substitution enables both the incorporation of range constrains on the model parameters and the assignment of Gaussian priors with the same variance for all the parameters. Here, $\min\{y_1\} = 10^{-6}$ and, for $j \geq 1$, $\min\{y_{2j}\} = 10^{-6}$, $\min\{y_{2j+1}\} = 1$, and $\max\{y_j\} = 10^3$. The model functions $F_k(\mathbf{Z},t_i)$ have only non-linear parameters.

As a result, marginalization over the linear parameters, as done by Bretthorst, was not necessary.

In the determination of numerical results, values of **Z** that maximize the posterior probability for **Z** given the data and the model are found using a simple simulated annealing routine [10, 11] followed by Newton's method [12]. Once the estimates of the parameters for each model are found, the posterior odds ratios are calculated as in [5].

Fig. 3 illustrates a decay curve found to have a single decay rate. The Schroeder decay curve in Fig. 3 was derived from data obtained in a church located in Starkville, Mississippi. For this decay curve, $\log_{10}[p(F_1|DI)/p(F_2|DI)] = 2.2$ and $T_1 = 1.16$ s. Fig. 4 illustrates a decay curve found to have two decay rates. The Schroeder decay curve in Fig. 4 was derived from data measured in a church located in Pascagoula, Mississippi. For this decay curve, $\log_{10}[p(F_2|DI)/p(F_1|DI)] = 3.2 \times 10^2$, $\log_{10}[p(F_2|DI)/p(F_3|DI)] = 1.5$, $T_1 = 0.36$ s and $T_2 = 0.99$ s.

SUMMARY

This paper presents a Bayesian method in architectural-acoustic applications for both selecting the number of sound energy decay rates and for estimating the values of the decay times from measured Schroeder decay functions. The method incorporates prior information regarding both the possible range of model parameters and the relationship between the parameters by using a re-parameterized version of an existing multi-exponential decay model. Because the presence of multiple energy decay rates can have a distinct impact on a hall's perceived acoustical quality, this work is of practical significance in architectural acoustics practice.

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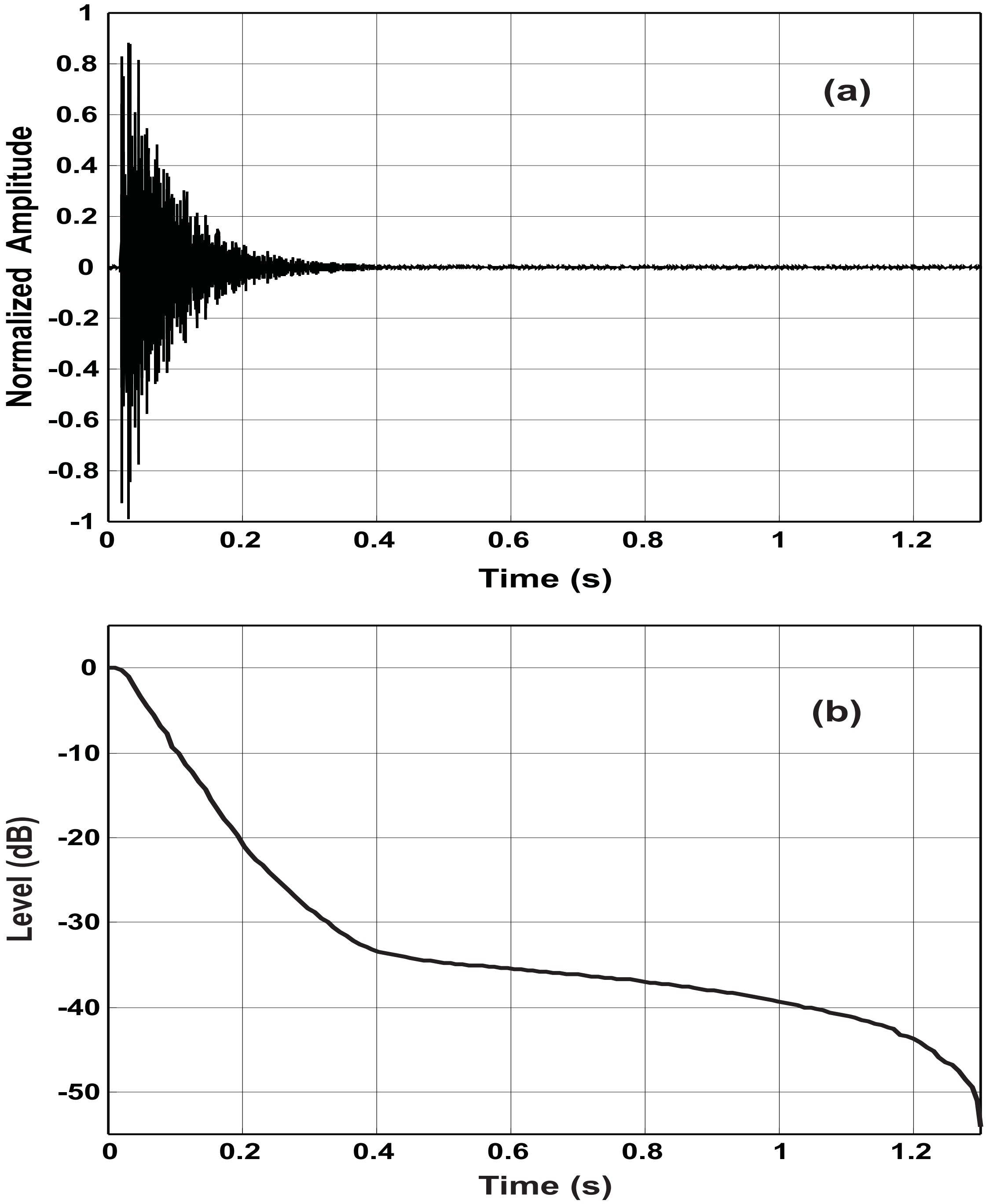
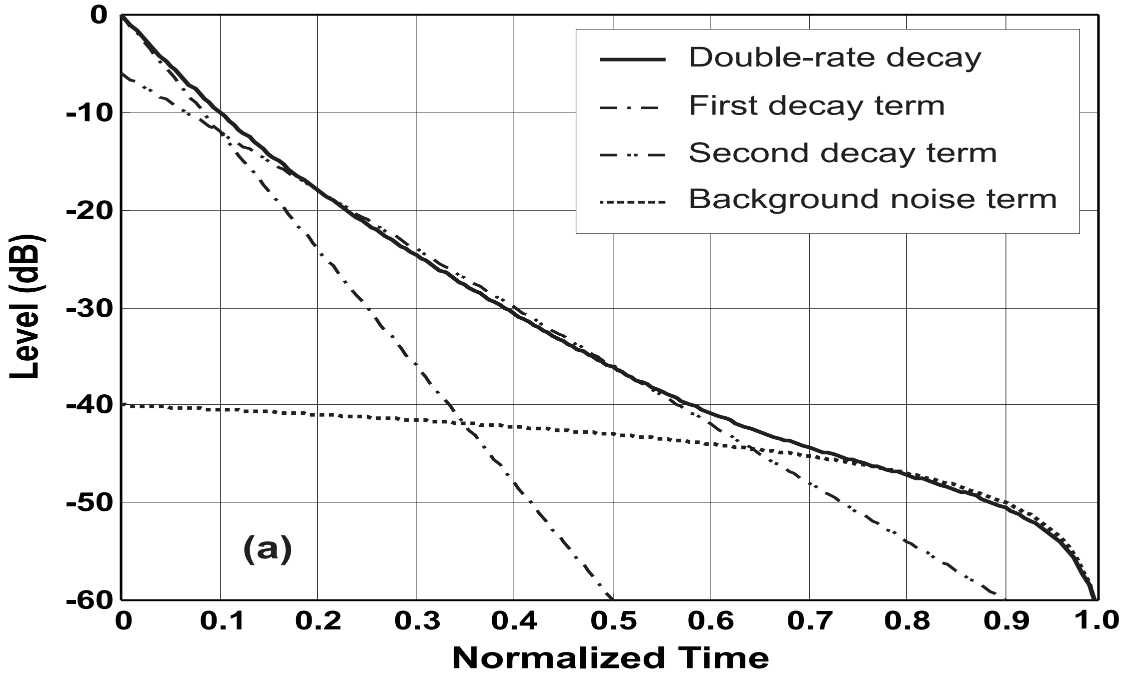


FIGURE 1. Room impulse response and Schroeder decay function determined from an experimentally measured room impulse response in two coupled scaled-down model rooms. (a) Room impulse response octave band-pass filtered at 1kHz with peak-to-noise ratio equal to 53 dB. (b) Schroeder decay function.



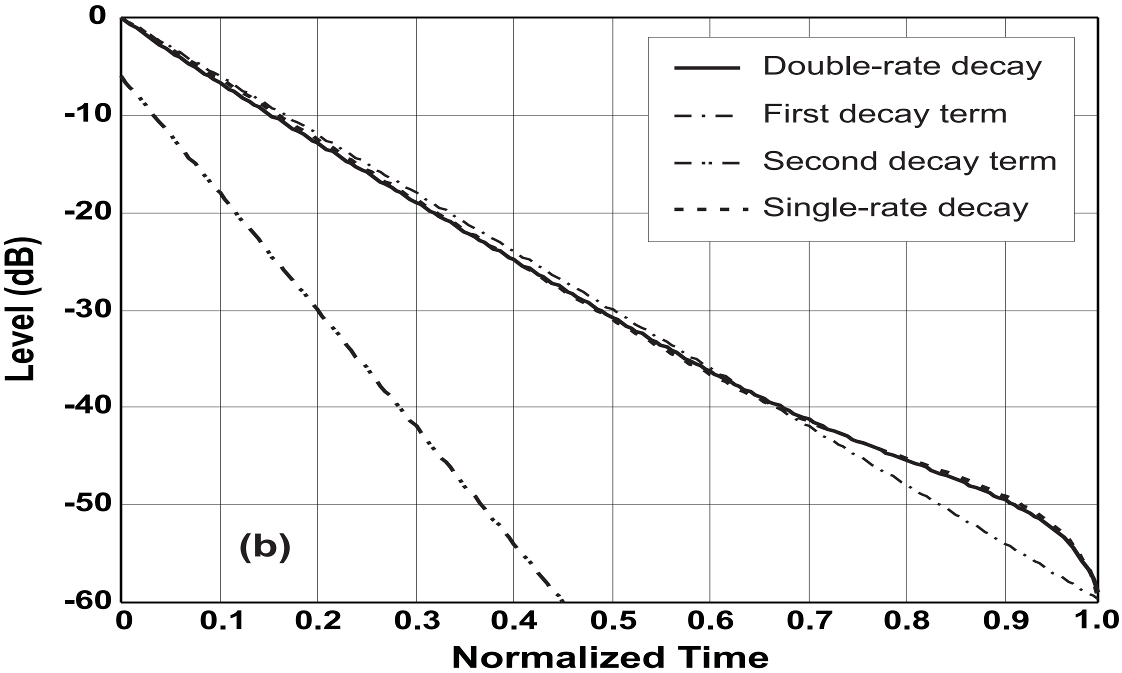


FIGURE 2. Schroeder decay functions simulated using a double-rate decay model. A normalized time scale is used for simplicity. (a) Decay parameters fulfill the conditions in (6) with $0.5 = T_1 < T_2 = 1.0$, $10\log_{10}(x_1) = 0$ dB, $10\log_{10}(x_3) = -6$ dB and $10\log_{10}(\langle n^2\rangle/N) = -45$ dB. (b) Decay parameters break the conditions in (6) with $1.0 = T_1 > T_2 = 0.5$, $10\log_{10}(x_1) = 0$ dB, $10\log_{10}(x_3) = -6$ dB and $10\log_{10}(\langle n^2\rangle/N) = -45$ dB. A single-rate decay curve with $T_1 = 0.96$ is also plotted for comparison. The double-rate decay curve in (b) is very close to the single-rate curve (from [2]).

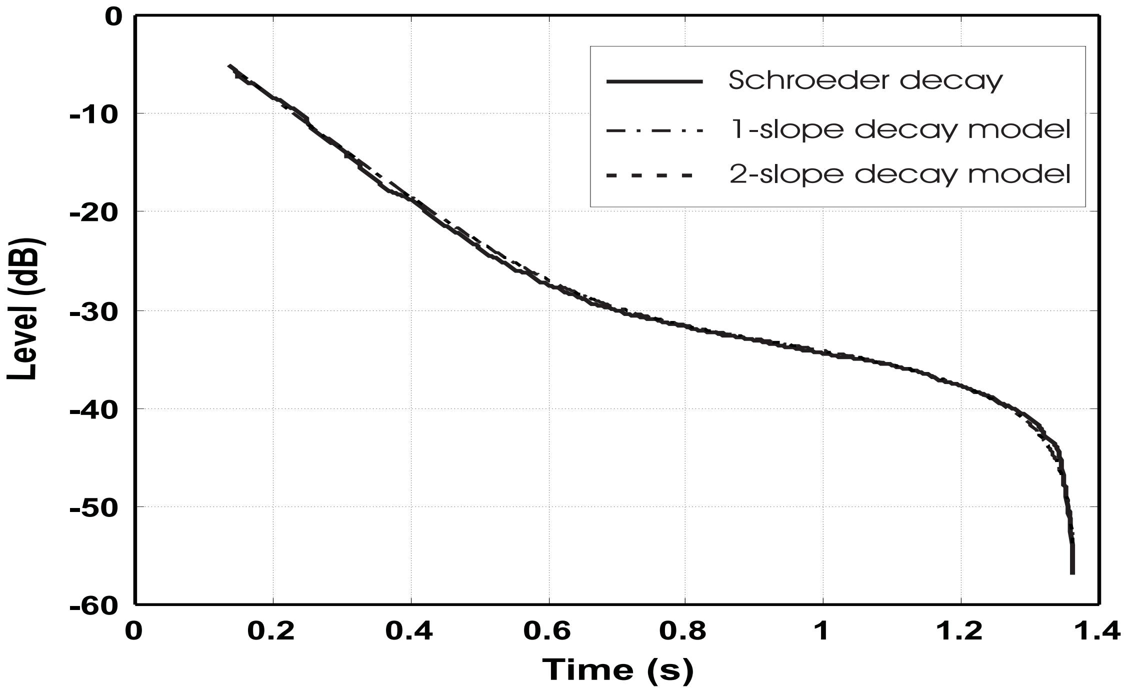


FIGURE 3. Measured Schroeder decay function for a church in Starkville, Mississippi in comparison with model functions determined by Bayesian estimation of decay parameters.

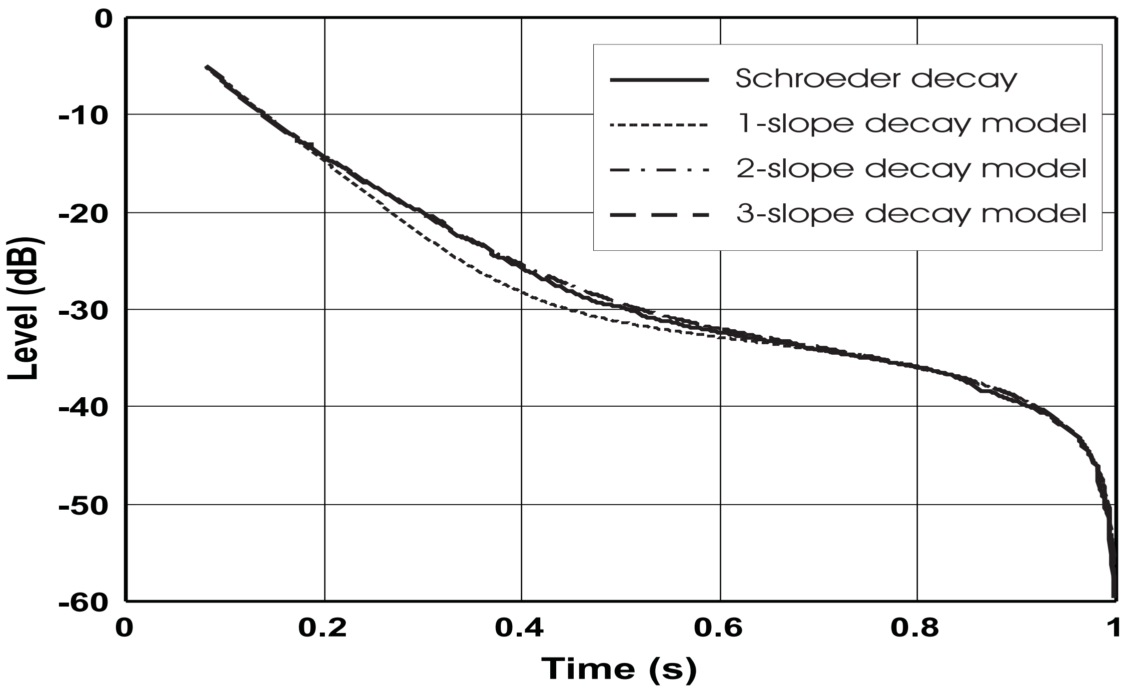


FIGURE 4. Measured Schroeder decay function for a church in Pascagoula, Mississippi in comparison with model functions determined by Bayesian estimation of the decay parameters.