Effect of variability of soil physical properties on clutter and false alarms in land mine detection

Vladimir N. Fokin, Margarita S. Fokina, James M. Sabatier, Ning Xiang, and Wheeler B. Howard
National Center for Physical Acoustics, University of Mississippi, University, Mississippi, USA

Received 31 July 2003; revised 17 March 2004; accepted 12 April 2004; published 3 August 2004.

The effects of variability of the ground on land mine detection and false alarm rates are analyzed within the framework of a viscoelastic-layered model of the ground. A matrix technique was used to describe sound interaction with layered viscoelastic ground. The resonance method in combination with a global search method is used to estimate a set of parameters for a three-layered viscoelastic ground model. Results of the estimation show good agreement between computed and experimental data. The effect of a finite size of sound source on the acoustic-to-seismic transfer function is discussed. The effect of variability of ground properties on the acoustic-to-seismic transfer function (admittance function) is analyzed. Analysis is performed on the plane of parameters of the layered ground in a wide frequency range for all angles of incidence. It is revealed that small variations in the shear speed in the top layer of ground will not cause variation in the acoustic-to-seismic transfer function at low frequencies but may cause strong variation at high frequencies. Results of outdoor measurements of the acoustic-to-seismic transfer function are presented, and a correlation between the high magnitude of the acoustic-to-seismic transfer function in certain frequency ranges and moisture content on the surface is revealed. A simple model explaining the correlation between moisture content in the upper layer, the acoustic-to-seismic transfer function, and ground properties is suggested.

INDEX TERMS: 0933 Exploration Geophysics: Remote sensing; 3210 Mathematical Geophysics: Modeling; 3260 Mathematical Geophysics: Inverse theory; KEYWORDS: sensing of land mines, natural ground variability, false alarms


1. Introduction

Investigations of environmental variability (and in particular the natural variability of ground) are closely connected with a number of applications. The most important are for noncontact methods of soil physical properties studies and land mine detection research. The natural ground variability has a strong effect on the land mine detection. Understanding the nature of acoustic false alarms and its connection with the environmental phenomenology will directly contribute to both increased probability of detection of a mine and/or a decreased rate of false-target alarms. These tasks require thorough investigations of ground variability.

Analysis of outdoor sound interaction with the ground has received considerable attention in the work of Embleton et al. [1977], Attenborough et al. [1986], Bass and Bolen [1980], Embleton et al. [1983], and Champoux and Stinson [1992], where a poroelastic model of the ground was explored. These studies have concentrated on sound propagation over the ground. Several authors have observed that an airborne acoustic wave incident on a ground surface could couple energy into the ground. These observations were based on measurements using geophones. Since the geophones are sensors that respond to the velocity of the medium, this signal was interpreted as being associated with the compressional wave in the elastic media [Sabatier et al., 1986; Albert and Orcutt, 1989, 1990]. A poroelastic model of the ground that considers propagation in the media of two types of compressional waves: “fast” and “slow” waves, and shear wave is the most adequate...
theory for the description of the ground. This poroelastic model is a complicated model with 14 input parameters for each poroelastic layer. Some of the parameters are well-known physical parameters, but others, such as air permeability, air porosity, and pore tortuosity, are difficult to measure in situ [Attenborough et al., 1986; Sabatier et al., 1996]. Because of this, it is difficult to properly apply and test this model.

[4] It is possible to consider acoustic-to-seismic coupling within the framework of a viscoelastic model because ground motion in the poroelastic model of the ground is associated with the “fast” compressional wave. There are investigations which show that the acoustic-to-seismic transfer functions (TF) predicted in the framework of the elastic and the poroelastic models are very close to each other [Harrop, 1999] when attenuation in the viscoelastic model is only slightly less than the attenuation in the poroelastic model. In comparison with the poroelastic model, the viscoelastic model needs six parameters for each viscoelastic layer. Therefore it is reasonable to use the simpler viscoelastic model in the initial effort to understand the effect of natural variability of the ground on the acoustic-to-seismic transfer function over a broad range of frequencies and sound angles of incident.

[5] The mathematical model for reflection of plane waves from the multilayered elastic media was developed in a number of papers [Gilbert and Backus, 1966; Thomson, 1950; Haskell, 1953; Frasier, 1970; Claerbout, 1968; Brekhovskikh and Godin, 1999; Fokin and Fokina, 2000] and was successfully used for predicting sound propagation and reflection. Specific computational schemes were discussed by Dunkin [1965], Thrower [1965], Schmidt and Jensen [1985], and Ivansson [1999]. While the equations for wave propagation in inhomogeneous elastic media have been considered by a number of authors [Ursin, 1983; Robins, 1994, 1998; Gupta, 1966], there are no investigations of the influence of different parameters of the ground on the acoustic-to-seismic transfer function in wide frequency and angular ranges.

[6] Another important problem is obtaining the properties of layered elastic media by acoustical means. Today, many methods for determining the properties of elastic media have been developed [Diachok et al., 1995; Dossou and Wilmot, 2000; Tolstoy and Chapman, 1998; Gerstoft, 1994]. These methods differ in procedure and the characteristics of acoustic signals used for obtaining the properties of the elastic media. Pulse methods based on measuring the characteristics of acoustic pulses and matched-field procedures are used most often [Fokin and Fokina, 2003].

[7] The determination of media parameters with the use of broadband acoustic signals also offers a possibility to improve the reliability of the results. Different characteristics of acoustic signals used for determining the properties of the media have different sensitivities to the parameters of the media. In this context, there is a need to investigate the characteristics that are sensitive to small variations of the media parameters. The resonance peaks of the acoustic-to-seismic transfer function can be qualified as such characteristics. Recently, the resonance approach has gained prominence, which is related to the development of computer methods and the progress in the theory of sound propagation [Breit and Wigner, 1936; Fiorito et al., 1981, 1979; Fokina and Fokin, 2001; Fokin and Fokina, 2001, 2003; Nagl et al., 1982].

[8] This paper deals with the effect of parameters in the layered viscoelastic ground on the acoustic-to-seismic transfer function. A matrix technique is used to describe sound interaction with the layered viscoelastic ground. The resonance approach is used for the estimation of a set of parameters for the ground model from experimental data. Analysis of experimental data obtained at an Army test site in Virginia allows one to determine the correlation between moisture content at the surface of the ground and to suggest a simple model explaining this phenomenon.

2. Physical Model and Mathematical Background

[9] The physical model used in the study of the acoustic-to-seismic transfer function consists of elastic layers covering an elastic half space (Figure 1). The parameters associated with the air and substrate are identified by subscripts 0 and \( \infty \); \( c_0 \) and \( \rho_0 \) are the speed of sound and the density in the air column; \( c_{ij}, c_{ij}', c_{ij''} \), and \( \rho_{ij} \) are the thickness, the velocities of the compression and shear waves, and the density in the \( j \)-th elastic layer; and \( c_{ij\infty}, c_{ij\infty}' \), and \( \rho_{ij\infty} \) are the velocities of the compression and shear waves, and the density in the elastic half space (substrate). The ground parameters are assumed to be constant within the elastic layer. The air column and the elastic half space are assumed to be homogeneous and semi-infinite. Attenuation effects are taken into account in the layers and in the half spaces assuming that the compression and shear waves velocities are complex values \( \tilde{c}_{ij} = c_{ij} + i \cdot \eta_{ij} \). This requires that the wave numbers be complex: \( k = (\omega/c_{ij})(1 - i \cdot (\eta_{ij}/c_{ij}))(1 + (\eta_{ij}/c_{ij})^2) \), where \( \omega = 2\pi f \) and \( f \) is the frequency. In a rectangular coordinate system \((x, y, z)\) the \( z \) axis is taken to increase positively upward and be normal to the layer plane. We assume that the displacement fields \( U \) can be written in the terms of the scalar and the vector potentials \( \varphi \) and \( \psi \) for the waves of vertical (SV waves) and horizontal (SH waves) polarizations:

\[
U = \nabla \varphi + \nabla \times \psi.
\]

(1)
When taking only waves of vertical polarization (SV waves) into account, the displacement fields are not dependent upon the y coordinate and do not contain a y component of the displacement $U = U(U_x, U_y = 0, U_z)$. $U$ is expressible as the sum of an irrotational component, which represents a compression wave, and a solenoidal component, which represents a shear wave. The vector potential $\psi$ represents the component of motion with nonzero vorticity. The reference axis of the system can be chosen such that $\psi$ has only one component along the $y$ axis; that is to say, $\psi = (0, \psi_y, 0)$. Stress and strain relations for a locally isotropic solid in a linear approach to determine the amplitude of strain are given by the generalized Hook’s law written in the tensor form:

$$\sigma_{ij} = \lambda \delta_{ij} U_{\gamma \gamma} + 2\mu U_{ij},$$

where $\sigma_{ij}$ is the stress tensor; $\lambda$ and $\mu$ are the Lamé's parameters; $\delta_{ij}$ is Kronecker’s symbol; $U_{\gamma \gamma} = \partial U_\gamma / \partial x + \partial U_\gamma / \partial z$ is the invariant, which characterizes the relative changing of element volume; and $U_{ij} = (1/2)(\partial U_i / \partial y + \partial U_j / \partial x)$ is the linearized strain tensor, which characterizes the increment of squared distance between two closely set points. Using equation (1) and the generalized Hook’s law (equation (2)), the normal and tangential components of the displacement and the stress tensor in isotropic media can be written in terms of the potentials $\varphi$ and $\psi_y$:

$$U_x = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi_y}{\partial z},$$

$$U_z = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi_y}{\partial x},$$

$$\sigma_{xz} = 2\mu \frac{\partial^2 \varphi}{\partial x \partial z} - \mu \frac{\partial^2 \psi_y}{\partial z^2} + \mu \frac{\partial^2 \psi_y}{\partial x^2},$$

$$\sigma_{zz} = (\lambda + 2\mu) \frac{\partial^2 \varphi}{\partial z^2} + \lambda \frac{\partial^2 \psi_y}{\partial x^2} + 2\mu \frac{\partial^2 \psi_y}{\partial x \partial z},$$

where $\lambda$ and $\mu$ are the Lamé parameters. The velocities $c_\ell$ and $c_t$ of the compression and shear waves are given by the well-known relations $c_\ell = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_t = \sqrt{\mu/\rho}$. The components of the displacement ($U_x, U_z$) and components of the stress tensor ($\sigma_{xz}, \sigma_{zz}$) involved in the boundary conditions are continuous across the elastic/elastic ($z = d_j$) and the air/elastic ($z = 0$) interfaces:

$$z = d_j,$$

$$\begin{cases}
U_{yj} = U_{yj+1}, \\
U_{zj} = U_{zj+1}, \\
\sigma_{xj} = \sigma_{xj+1}, \\
\sigma_{zj} = \sigma_{zj+1},
\end{cases}$$

where $z = 0,$

$$\begin{cases}
U_{z0} = U_{z1}, \\
0 = \sigma_{x1}, \\
-p_0 = \sigma_{z1}.
\end{cases}$$

The potentials $\varphi$ and $\psi_y$ are governed by the Helmholtz equations:

$$\Delta \varphi + \alpha^2 \varphi = 0,$$

$$\Delta \psi_y + \beta^2 \psi_y = 0,$$

where $\alpha^2 = k_t^2 - \xi^2$ and $\beta^2 = k_s^2 - \xi^2$ are the squares of the vertical components of the wave numbers for the compression and shear wave velocities, $k = \omega/c_\ell$, $k_s = \omega/c_s$, $\xi = k \cdot \sin \theta_0 = k_t \cdot \sin \theta_t = = k_t \cdot \sin \theta_0$ are the wave numbers and the propagation constant, and $\theta_0$ is the incidence angle of the plane wave. Assuming a plane wave incident in the air, the solutions of equation (6) can be written in the terms of the potentials describing the compression and shear waves:

$$\varphi(z) = \varphi^+ e^{i\alpha z} + \varphi^- e^{-i\alpha z},$$

$$\psi_y(z) = \psi^+ e^{i\beta z} + \psi^- e^{-i\beta z},$$

where $\varphi^+, \varphi^-, \psi^+, \psi^-$ are the some arbitrary functions that characterize the elastic waves propagating in the positive (with the superscript $+$) and negative (with the superscript $-$) direction of the $z$ axis. For an incident plane wave of unit amplitude, which excites the given

---

**Figure 1.** Physical model for modeling of the acoustic-to-seismic transfer function.
elastic layered system, the depth dependence of the potentials in the air and elastic half spaces are given:

\[
\varphi_0(z) = \varphi_0^+ e^{i\omega z} + \varphi_0^- e^{-i\omega z},
\]

(8)

\[
\psi_{j0}(z) = 0,
\]

\[
\varphi_\infty(z) = \varphi_\infty^+ e^{i\omega z} + \varphi_\infty^- e^{-i\omega z},
\]

(9)

\[
\psi_{\infty}(z) = \psi_\infty^+ e^{i\omega z} + \psi_\infty^- e^{-i\omega z},
\]

where \(\varphi_0^- = R\) is the reflection coefficient; \(\varphi_0^+ = 1\) is the amplitude of the plane wave under the angle of incidence \(\theta_0\) in the air half space; and \(\varphi_\infty^+, \varphi_\infty^-, \varphi_\infty^+, \varphi_\infty^-\) are the amplitudes of the compression and shear waves in the elastic half-space, respectively. Here \(\varphi_\infty = W_t\) and \(\psi_\infty = W_s\) are the refraction indices of the compression and shear waves in the substrate. Taking into account the radiation conditions at infinity, then \(\varphi_\infty = 0\) and \(\psi_\infty = 0\). Substituting equations (7) and (9) in the boundary condition (5) across the elastic/elastic \((z = d_j)\) interfaces and performing the differentiation at the boundaries, we obtain \(4(n+1)\) equations in \(4(n+1)\) unknowns \(\varphi_j, \psi_j, \varphi_{j+1}^+, \psi_{j+1}^+\).

[10] We will characterize the displacement field not by the column vector potential \(\vec{\varphi}(z) = (\varphi_j, \psi_j, \varphi_{j+1}^+, \psi_{j+1}^+)^T\) (\(T\) denotes the operation of transposition), which in a complicated manner transforms across boundaries, but by the column vector \(\vec{f}(z) = (U_x, U_y, \sigma_{xz}, \sigma_{zz})^T\). The vector \(\vec{f}(z)\) by virtue of the boundary conditions is continuous, as opposed to the vector potential \(\vec{\varphi}(z)\):

\[
\vec{f}(z) = \begin{bmatrix} U_x \\ U_y \\ \sigma_{xz} \\ \sigma_{zz} \end{bmatrix} = A_j \cdot \vec{L}_j \cdot \vec{f}(z)
\]

(10)

Using the constancy of the column vector \(\vec{f}(z)\) inside the layer and in equation (10), it is easy to obtain the required relation between \(\vec{f}(z)\) and \(\vec{f}(z_{j+1})\) at two adjoined boundaries \(j\) and \(j + 1\) [Brekhovskikh and Godin, 1989]:

\[
\vec{f}(z) = A_j \cdot \vec{L}_j \cdot \vec{f}(z_{j+1}).
\]

(11)

Here \(A_j\) and \(A_j^{-1}\) are the right and invert matrices of the elastic layer, and \(L_j\) is the diagonal matrix for interlayer contacts:

\[
A_j = \begin{bmatrix} i\xi & i\xi & -i\beta_j & i\beta_j \\ i\alpha_j & -i\alpha_j & i\beta_j & i\beta_j \\ 2\mu_j \xi \lambda_j & 2\mu_j \xi \lambda_j & -2\mu_j \xi \beta_j & 2\mu_j \xi \beta_j \\ 2\mu_j \xi \alpha_j & 2\mu_j \xi \alpha_j & -2\mu_j \xi \alpha_j & 2\mu_j \xi \lambda_j \end{bmatrix}.
\]

(12)

The consecutive application of equation (11) to \(j = 1, \ldots, n\) layers and interlayer contacts permits connection of the column vector between the boundary of first and second layers \(\vec{f}(z_1)\) and the value at the boundary \(n\)th layer and elastic half space \(\vec{f}(z_\infty)\):

\[
\vec{f}(z_1) = D \cdot \vec{f}(z_\infty),
\]

(14)

where \(D = A_1^{-1} \cdot A_2 \cdot L_2 \cdot A_2^{-1} \cdots A_n \cdot L_n \cdot A_n^{-1} \cdot A_\infty\) is matrix propagator characterizing the viscous elastic layered half space and elastic half space; and \(A_\infty\) is the characteristic matrix of the elastic half space, which looks like matrix \(A_j\) in equation (12), but all material parameters use the subscript \(\infty\). If we know the elements of matrix \(D\) and incident plane wave perturbation \(\varphi_0^+ = 1\), then for the combined description of the upper air half space and multilayered elastic media, one can write six boundary equations:

\[
\begin{align*}
U_x &= d_{11}\varphi_\infty^+ + d_{12}\varphi_\infty^- + d_{13}\psi_\infty^+ + d_{14}\psi_\infty^- \\
U_y &= d_{21}\varphi_\infty^+ + d_{22}\varphi_\infty^- + d_{23}\psi_\infty^+ + d_{24}\psi_\infty^- \\
0 &= d_{31}\varphi_\infty^+ + d_{32}\varphi_\infty^- + d_{33}\psi_\infty^+ + d_{34}\psi_\infty^- \\
\sigma_{xz} &= d_{41}\varphi_\infty^+ + d_{42}\varphi_\infty^- + d_{43}\psi_\infty^+ + d_{44}\psi_\infty^- \\
\varphi_0^+ &= q_{11}U_x + q_{12}\sigma_{zz} \\
\varphi_0^- &= q_{21}U_x + q_{22}\sigma_{zz},
\end{align*}
\]

(15)

\[
Q^{-1} = \begin{bmatrix} 1 & -d_{10} & -d_{20} & -d_{30} & -d_{40} \\ -d_{10} & 1 & -d_{11} & -d_{12} & -d_{13} \\ -d_{20} & -d_{11} & 1 & -d_{21} & -d_{22} \\ -d_{30} & -d_{12} & -d_{21} & 1 & -d_{31} \\ -d_{40} & -d_{13} & -d_{22} & -d_{31} & 1 \end{bmatrix},
\]

(16)

where \(q_{km}(k = 1, 2, m = 1, 2)\) are the elements of the matrix \(Q^{-1}\) for the air half space, and \(d_{ij}(i = 1, \ldots, 4, j = 1, \ldots, 4)\) are the elements of the matrix propagator \(D\) for the layered viscoelastic half space. The invert matrices \(Q^{-1}\) for air half space are written as:

\[
Q^{-1} = \begin{bmatrix} 1 & -\frac{1}{2i\alpha_0} & -\frac{1}{2\omega^2\rho_0} \\ -\frac{1}{2i\alpha_0} & 1 & -\frac{1}{2\omega^2\rho_0} \\ -\frac{1}{2\omega^2\rho_0} & \frac{1}{2i\alpha_0} & 1 \end{bmatrix},
\]

\[
\omega = 2\pi f, \quad \alpha_0 = \sqrt{k^2 - \xi^2} \quad \text{is the vertical component of the wave number, and} \quad k = \omega/c_0, \quad c_0 \quad \text{is the sound speed in air.}
\]

[11] For solving the system of linear algebraic equation (15) with respect to the reflection coefficient \(\varphi_0^+ = R\), the refraction indices for compression \(\varphi_\infty^+ = W_t\) and shear waves \(\psi_\infty = W_s\), one may use the Cramer’s rule taking into account the radiation conditions at infinity.
\( \varphi_\infty = \psi_\infty = 0 \) and absence of the incident and reflected shear waves in the air half-space (\( \psi_0 = \varphi_0 = 0 \)):

\[
R = \frac{q_{21}(d_{21}d_{33} - d_{23}d_{31}) - q_{22}(d_{31}d_{43} - d_{41}d_{33})}{q_{11}(d_{21}d_{33} - d_{23}d_{31}) - q_{12}(d_{31}d_{43} - d_{41}d_{33})},
\]

(17)

\[
W_i = \frac{d_{33}}{q_{11}(d_{21}d_{33} - d_{23}d_{31}) - q_{12}(d_{31}d_{43} - d_{41}d_{33})},
\]

(18)

\[
W_i = \frac{-d_{31}}{q_{11}(d_{21}d_{33} - d_{23}d_{31}) - q_{12}(d_{31}d_{43} - d_{41}d_{33})}.
\]

(19)

If the complex pressure \( P \) and the reflection coefficient \( R \) are known, the normal particle velocity on the boundary with the solid layer \( V_z \) may be determined. The impedance \( Z_{in}(f, \theta) \) and the acoustic-to-seismic transfer function \( TF(f, \theta) \) may be expressed as:

\[
Z_{in}(f, \theta) = \frac{P}{V_z} = \frac{\rho c_0}{\cos \theta} \cdot \frac{(1 + R(f, \theta))}{(1 - R(f, \theta))},
\]

(20)

\[
TF(f, \theta) = \frac{1}{Z_{in}(f, \theta)}.
\]

(21)

To increase computational accuracy, matrices of the fourth order should be replaced by the sixth-order matrices. A matrix technique using the sixth-order matrices was implemented as a functioning computer code to obtain \( TF(f, \theta) \). Results of test computations of this code are presented in the work of Fokina and Fokin [2000].

### 3. Effect of Distributed Sound Source

[12] In the source-receiver geometrical configurations used for land mine detection, the size of the sound source is often comparable with the distance to the point of measurement. To evaluate the effect of a distributed sound source on the acoustic-to-seismic transfer function, one may analyze \( TF(f, \theta) \) on the frequency-angle plane. The acoustic-to-seismic transfer function \( TF(f, \theta) \) for a typical set of ground parameters calculated in a wide frequency band \( f = 1–800 \text{ Hz} \) for all angles of incidence \( \theta = 0–90^\circ \) is shown in Figure 2 in the form of contours. Computations were performed for a single elastic layer covering an elastic half space. The color represents the amplitude of \( TF(f, \theta) \). The red color corresponds to maximum amplitude, while the blue color corresponds to minimum amplitude. The typical ground parameters were taken from Albert [1993], and their values were used in the computations shown in Table 1. The \( TF(f, \theta) \) on Figure 2 exhibits several sequences of minima and maxima, which are connected with resonances in the acoustic-to-seismic transfer function. Positions of maxima depend upon frequency, angle, thickness of the layer, and values of the compressional and shear speeds. The three critical angles existing in this layered model may be found in Figure 2. Critical angles associated with the compressional and shear waves in the half space (\( \theta_{l\infty} = 70.81^\circ \), \( \theta_{s\infty} = 11.53^\circ \)) are clearly seen in all frequencies. The critical angle connected with ground layering is not clearly observed in the low-frequency range. The reason is that the sound wavelength at low frequencies is much greater than the thickness of the layer (\( \lambda \gg d \)). At higher frequencies, critical angles connected with the layering of the ground are clearly seen. In the vicinity of the critical angle connected with the compressional velocity in the layer (\( \theta_l = 58.21^\circ \)) the extremes of the acoustic-to-seismic transfer function tend to be parallel to the frequency axis.

[13] As a first rough approximation, the sound field at the point of measurement may be approximated as a function of the source aperture over a range of corresponding incidence angles. Integration of the plane

### Table 1. Physical Properties of the Single-Layered Model

<table>
<thead>
<tr>
<th>Media</th>
<th>( c_r ), m/s</th>
<th>( \eta_r ), m/s</th>
<th>( c_s ), m/s</th>
<th>( \eta_s ), m/s</th>
<th>( \rho ), kg/m³</th>
<th>( d ), m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>340</td>
<td>0.017</td>
<td>0</td>
<td>0</td>
<td>1.225</td>
<td>∞</td>
</tr>
<tr>
<td>Elastic layer</td>
<td>400</td>
<td>2.6</td>
<td>160</td>
<td>2.6</td>
<td>1800</td>
<td>1</td>
</tr>
<tr>
<td>Elastic half space</td>
<td>1700</td>
<td>1.133</td>
<td>360</td>
<td>0.6</td>
<td>1800</td>
<td>∞</td>
</tr>
</tbody>
</table>

![Figure 2. Calculated acoustic-to-seismic transfer function (admittance function) on an angle-frequency plane.](image-url)
wave \( TF(f, \theta) \) over the range of incidence angles \( \theta_0 \) will give the \( TF(f) \) for the distributed source. Analyzing the dependence of the transfer function as a function of angle on Figure 2, one may notice that integration with respect to the angle will have practically no effect on the transfer function in the range of incident angle \( 0^{\circ} \)–11.53° and 58.21°–90°. This influence may be strong if the range of integration of the TF includes angles between critical angles \( \theta_{c1} = 11.53^{\circ} \) and \( \theta_{c2} = 58.21^{\circ} \). In this case, peaks of the transfer function will be lower and wider than peaks of the calculated TF for a single angle. This may be a reason why experimentally measured transfer functions often have wider peaks than calculated ones. Influence of integration on the TF strongly depends upon the set of parameters for the layered ground and may vary within very wide limits.

4. Estimation of the Parameters of a Layered Ground

[14] In this section the possibility of using a viscoelastic model for estimation of physical parameters of the ground is considered. A short description of experimental measurements and techniques used for obtaining parameters of the acoustic-to-seismic transfer function is presented.

[15] The acoustic-to-seismic \( TF \) was measured at a ground position located on the Mississippi Agriculture and Forestry Experimental Station, 7 miles north of Holly Springs, Mississippi [Howard, 2002]. Terrain in this area consists of grassy, low, rolling hills. Measurements were made on an uphill location, perpendicular to the hillslope at five different spots on the ground. A loudspeaker with an essentially flat frequency response over a range of 50–500 Hz was used as the sound source. Collocated geophones and microphones were used to measure the ratio of the normal component of the surface particle velocity to the acoustic pressure. To estimate the ground parameters, a layered viscoelastic model of ground consisting of two viscoelastic layers overlaying a viscoelastic substrate was used to model the measured TF at the fourth experimental point. The TF at this point has a good signal/noise ratio and a regular frequency structure. A trenched wall or exposed vertical slice of the near surface soil in the vicinity of the TF measurement site shows there are soil and fragipan layers over the water table. Measurements performed in the vicinity of this site include seismic measurements. These measurements provide approximate values of the same acoustical parameters and thicknesses of soil and fragipan layers. These parameters are shown in Table 2.

[16] To estimate acoustical parameters that provide the best fit to the experimental data, a two-stage procedure was used. For the first stage a resonance approach was applied. The main idea of this approach is that the position, half width, and amplitude of resonance in the reflection coefficient or the acoustic-to-seismic transfer function have an unequivocal link with the parameters of the layered viscoelastic model. The analytical formulas which permit one to obtain parameters of the media through measurements of the characteristics of resonances was obtained by Fiorito et al. [1979, 1981] for a model of a liquid layer laying over the liquid half space and for an elastic layer between two liquids. For the model of an elastic layer covering an elastic half space, the resonance expression for the reflection coefficient and expressions for resonance characteristics were obtained from Fokina and Fokin [2001]. Owing to the extreme awkwardness of the resonance expressions, even for one viscoelastic layer covering an elastic half space, it is very difficult to obtain such expressions for a two-layered viscoelastic model. In this section a modified resonance approach was developed and used for obtaining properties of the viscoelastic media. This study used only resonance positions in the acoustic-to-seismic transfer function to determine a set of acceptable parameters of the media, or in other words, for quickly rejecting the sets of data that can not fit the experimental data due to improper positions of the resonances. Because the same resonance positions may be obtained for different combinations of parameters of layered media, only the resonance positions at first stage of reconstruction were used. The sets of data with proper positions of the resonance extremes were then used to search for sets of data with the best fit of experimental data when all other parameters of the media were varied. Practically, one can use the coincidence of frequency resonance positions as indication of proximity of calculated and measured transfer functions.

[17] The good agreement between calculated and measured transfer functions is only possible when the positions of the main extremes in these two functions are approximately the same. Analysis of the sensitivity of the acoustic-to-seismic TF to different parameters of the model show that the positions of resonance minima and maxima in the TF are insensitive to parameters of the elastic half space, densities of the layers, and atten-

---

### Table 2. Parameters of Media Obtained From Seismic Measurements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First layer</td>
<td></td>
</tr>
<tr>
<td>( c_{11} )</td>
<td>140–180 m/s</td>
</tr>
<tr>
<td>( \rho_1 )</td>
<td>1300–1600 kg/m³</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>0.45–0.6 m</td>
</tr>
<tr>
<td>Second layer (Fragipan)</td>
<td></td>
</tr>
<tr>
<td>( c_{12} )</td>
<td>337–530 m/s</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>200–230 m/s</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>1560 kg/m³</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>1.32 m</td>
</tr>
</tbody>
</table>
vation of compressional and shear waves. In the first stage of reconstruction, when one is interested only in resonance positions, the amplitudes of the minima and maxima are unimportant, and this reduces the number of parameters to be estimated. The number of parameters in the model may be reduced from 17 to 6. These six parameters are $c_f$, $c_p$, and $d$ for first and second layers. Furthermore, if only the frequencies of the maxima and minima are of interest, the TF is required only in the vicinity of these frequencies. It is possible to calculate the model of the TF in the vicinity of the minima or maxima and to check for the existence of the required extrema. It may be necessary to check for the existence of other frequency extremes of the TF. In many cases, checking the position of just one resonance is enough to determine that this model is not good enough. This procedure allows for an increase in the computational speed of the models of up to 100 times when performing a global search for the six most important parameters.

[18] As a result of a global search, a set of data, which has proper positions of six resonances, was obtained. The set of data was obtained from approximately 50,000 combinations of six parameters of interest. To find the model with the best fit to the experimental data, the mean square difference between the experimental data and these models was calculated in the whole frequency band. Additional variation of all other parameters of the model was applied, and the effect of a distributed sound source was also taken into account. Figure 3 shows both the experimentally measured transfer function (red curve) and the curve with best fit to the experimental data (blue). Parameters obtained as a result of this estimation are shown in Table 3.

[19] Figure 3 shows that the most significant features of the experimentally measured transfer function are presented on the computational curve. Differences between these curves for frequencies less than 70 Hz are connected with the fact that the geophone is sensitive to airborne sound at low frequencies [Howard, 2002]. The experimental curve should be lower at these frequencies. Overall, there is good agreement between the calculated and measured TFs. Comparisons between calculated and measured transfer functions make it possible to determine that the small-scale modulation in the calculated and measured TF is connected with shear wave speed in the first layer. The frequencies and magnitude of the maxima in the small-scale modulation in the TF at 90 and 250 Hz are in very good agreement with the experimental curve. This agreement decreases in the frequency ranges 160–210 and 420–500 Hz; however, even in these frequency ranges, the frequencies of the minima and maxima in the calculated curve coincide with the frequencies of minima and maxima in the experimental curve. Parameters of the ground determined through the inversion procedure lay within the acceptable physical limits and are close to parameters measured by other methods. Differences in the calculated and experimental curves in the same frequency ranges are an indication of the necessity of using a more complicated ground model for the acoustic-to-seismic transfer function. The compressional sound speed obtained in the top layer as the result of inversion is less than the speed obtained by means of seismic measurements. This may be connected with the fact that the wavelength of the compressional sound speed used in seismic measurements is close to the depth of the first layer. As a result, higher values of sound speed may be obtained by the seismic method. The good agreement between calculated and measured TF confirms that the viscoelastic model of

Table 3. Parameters of Media Determined Through the Acoustic-to-Seismic Transfer Function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air half space</td>
<td>$c_0 = 340 \text{ m/s}$, $\rho_0 = 0.017 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$\rho_0 = 1.225 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>First layer</td>
<td>$c_{f1} = 96 \text{ m/s}$, $\eta_0 = 3.5 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$c_{l1} = 72 \text{ m/s}$, $\eta_{f1} = 0.7 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$\rho_1 = 1532 \text{ kg/m}^3$, $d_1 = 0.59 \text{ m}$</td>
</tr>
<tr>
<td>Second layer (Fragipan)</td>
<td>$c_{f2} = 560 \text{ m/s}$, $\eta_2 = 6.5 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$c_{l2} = 185 \text{ m/s}$, $\eta_{f2} = 1.8 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$\rho_2 = 1560 \text{ kg/m}^3$, $d_2 = 1.3 \text{ m}$</td>
</tr>
<tr>
<td>Half space (water table)</td>
<td>$c_{w} = 1800 \text{ m/s}$, $\eta_{w} = 2.0 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$c_{w} = 500 \text{ m/s}$, $\eta_{w} = 2.7 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td>$\rho_{w} = 2400 \text{ kg/m}^3$</td>
</tr>
</tbody>
</table>
the ground may be used for investigations effect of
ground characteristics on the acoustic-to-seismic TF.

5. Effect of Ground Parameters Natural Variability

[20] Owing to natural variability, all ground parameters have both regular spatial dependence and random spatial fluctuations. Sensitivity of the TF to different parameters of the ground varies. The amplitude of the TF has a high sensitivity to attenuation in the ground. Positions of the extremes in the acoustic-to-seismic transfer function have high sensitivity to values of compressional and shear sound speeds and low sensitivity to attenuation in the ground and density. Investigation of dependencies of the acoustic-to-seismic transfer function versus different parameters of layered elastic media may be used for understanding false alarms in land mine detection. In this section the influence of the same parameters of media on the acoustic-to-seismic TF will be considered.

[21] Numerous measurements of the acoustic-to-seismic TF performed at different sites and outdoor conditions for detection of land mines show that the magnitude of TF has strong spatial variability [Xiang and Sabatier, 2003]. This variability is thought to be connected with the natural variability of the ground. The variability of the TF at low frequencies is shown to be minimal and increases dramatically with increasing frequency. The acoustic-to-seismic transfer function measured over the same region of ground has practically no spatial variability at frequencies less than 100 Hz but has a strong spatial variability at frequencies above 300 Hz. This means that the same spatial fluctuations of ground parameters should have no effect on the transfer function at low frequencies yet should have a strong effect on the transfer function at high frequencies. Since the majority of antipersonnel mines have maximum responses for frequencies higher than 300 Hz, this effect essentially increases the false alarm rate for antipersonnel mines. An understanding of the reasons that cause the variability of magnitude of TF from one spatial point to another will help to eliminate this influence. To understand the effect of the natural variability of ground parameters on the acoustic-to-seismic TF, numerical investigations were performed for a model consisting of one elastic layer overlaying an elastic half space.

[22] Numerical investigations show that parameters of the elastic half space have less effect on the acoustic-to-seismic TF than the thickness of the layer and compressional and shear speed in the layer. Here we will consider the effect of the shear wave speed in the layer and the thickness of the layer on the acoustic-to-seismic TF. It is convenient to examine the TF on the frequency shear speed plane (Figures 4a and 4b) or on the frequency thickness of the layer plane (Figure 5). During variation of one selected parameter, all other parameters of the model were kept constant. Sets of parameters used for computations are shown in Table 4.

[23] A vertical line section or slice of the surface in Figure 4a parallel to the frequency axis will give the dependence of the TF upon frequency with the selected value of the shear speed. A slice of Figure 4a parallel to the shear speed axis will give the dependence of the TF upon shear speed for the selected frequency (Figure 4b).

[24] Minima and maxima of the TF in Figure 4a form a set of the fan-like straight lines coming from the point with the coordinates (0,0). Analysis of TF\( (f, c) \) in Figure 4a makes it possible to explain why the number of false
indicators increases when the frequency increases [Sabatier and Xiang, 2001]. For a 1 m² outdoor soil plot, let $c_t$ have a range of variability 90–100 m/s from one spatial point to another. At low frequency ($f = 50$ Hz), changing $c_t$ does not lead to significant changes in the TF. A horizontal section of the surface in Figure 4a at 50 Hz shows that the value of the TF does not significantly change for this range of shear speeds. However, in a higher-frequency range, e.g., at 800 Hz, the TF value changes from a maximum to a minimum twice within the same variation of $c_t$ in the layer. This means that small variations of $c_t$ will be more likely to cause fluctuations in the TF at high frequencies than at low frequencies.

If one takes sections of $\text{TF}(f, c_t)$ for frequencies of 52, 350, 550, and 800 Hz (Figure 4b), it is clearly seen that the TF at $f = 52$ Hz (red line) has no variations for $c_t > 30$ m/s, while the TF at $f = 800$ Hz (magenta line) has strong variations for $c_t > 30$ m/s. Figure 4a shows that at higher frequencies, a greater number of oscillations occur in the TF. The probability that variations of shear speed will be in the vicinity of a peak of the TF is greater at higher frequencies than at lower frequencies. In the vicinity of a peak in the TF, small variations of the shear speed will lead to significant changes in the magnitude of the TF. For example, in the vicinity of a peak (magenta line, $c_t = 100$ m/s) m decreasing the value of $c_t$ by 4% will increase the magnitude of the TF by 17 dB.

Dependence of the acoustic-to-seismic transfer function versus the thickness of the layer illustrates that frequency position of first maximum in the acoustic-to-seismic transfer function strongly depends upon the thickness of the layer (Figure 5). The position of the maximum shifts to the lower frequencies for the thick layers and shifts to the higher frequencies for the thin layers. If the thickness of the layer varies in the space, it is reasonable to expect that regions with smaller thickness layers will have a response at higher frequencies than regions with thicker layers. For example, the position of the maximum shifts from 175 to 350 Hz when the thickness of the layer changed from 0.01 to 0.2 m. Assuming that all other parameters of the model are fixed (Table 4), displacement of first maxima of the transfer function may be used as an indicator of changes in the thickness of the layer. This feature of the TF will be useful if (in same spatial spots) the properties of the upper layer are different from properties of the surrounding area. According to numerical investigations, when the smaller density is combined with a smaller sound speed in the top layer, these spots will have a larger value of the transfer function. Therefore they can be easily monitored. The spatial dependence of the frequency of the first maximum in the transfer function will describe spatial variance in layer depth inside these spots or between different spots with similar properties.

A sensitivity analysis of the TF shows that the probability of fluctuations in the magnitude of the TF due to natural variability of the ground over the same region of the ground will be greater at high frequencies than at low frequencies. It was shown that frequency position of the maxima of the transfer function may be used for monitoring depth of surface layer of the ground.

6. An Analysis of Experimental Data of Clutter and Strong False Indicators

In this section, analysis of experimental data of natural variability of ground is presented. The experimental investigations were performed at an Army test site in April 2003. The test site consisted of several square meters of a constructed gravel road described elsewhere [Xiang and Sabatier, 2003]. The acoustically induced vibration of the ground was measured with a scanning laser Doppler vibrometer (LDV) interrogating 480 × 480 mm areas using a 1.25 Hz frequency resolution over the frequency band of 100–1000 Hz. The

<table>
<thead>
<tr>
<th>Media</th>
<th>$c_s$, m/s</th>
<th>$\eta_s$, m/s</th>
<th>$c_n$, m/s</th>
<th>$\eta_n$, m/s</th>
<th>$\rho$, kg/m</th>
<th>$d$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>340</td>
<td>0.017</td>
<td>0</td>
<td>0</td>
<td>1.225</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Layer</td>
<td>200</td>
<td>3.5</td>
<td>70</td>
<td>1</td>
<td>1400</td>
<td>1.32</td>
</tr>
<tr>
<td>Half space</td>
<td>1800</td>
<td>3.5</td>
<td>500</td>
<td>2.2</td>
<td>2400</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Figure 5. Calculated acoustic-to-seismic transfer function: on frequency thickness of layer plane. All other parameters of two-layered viscoelastic model of the ground are fixed (Table 4).
measurements were performed in sunny, calm weather following a rainy day. Results of measurements on a blank site and a site with a mine are presented in Figures 6a and 6b. This blank site (Figure 6a) was chosen because the magnitude of the normal particle velocity in the center of the image has the same value as the magnitude of the response for the antipersonnel mine in Figure 6b (approximately $1.0 \times 10^{-7}$ m/s). It should be noted that magnitudes for the normal component of velocities of the ground surface vary over very wide ranges and strongly depend on the acoustical power of the loudspeaker and ground properties.

[29] To determine if a mine exists in an area scanned with an LDV, three main characteristics of the acoustic-to-seismic TF have been used [Rosen et al., 2000]:

[30] The magnitude of the TF on measurement points directly over the buried mine should be greater than the other measurement points.

[31] This higher-velocity response should have a circular shape (mines often have a cylindrical shape) approximately the size of a mine.

[32] The shape and position of the high-velocity spot in the scanned image should be unchanged over several frequency subbands.

[33] All of these characteristics are presented in the blank spot image. As such, it may be considered a false alarm for acoustic mine detection even though the shape of the response is not circular. There are well-known wooden rectangular mines that have TF images quite similar to the image in Figure 6a [Xiang and Sabatier, 2003]. The frequencies of the TF for which the mine and blank spots were observed are different, but these frequencies lie in the frequency band in which antipersonnel mines have strong velocity responses. The high-velocity response for measurement points on the blank scan area exists in the frequency band from 480 to 580 Hz. The magnitude of the coherence function between the measured particle velocity and the sound pressure level in the frequency band 400–600 Hz is equal to unity.

[34] At the time of the measurements the surface of the ground within the measurement site visually consisted of wet and dry spots of irregular shapes as seen in the background charged couple device (CCD) camera photo (Figure 7a). The reader must be careful not to confuse shadows of cables and equipment support structures with the light and dark spots associated with ground moisture. Positions of measurement points within the scan area with high particle velocities vary from one frequency subband to another and seem to have random locations. Analysis of LDV data in the frequency range 420–650 Hz revealed that measurement points with high particle velocity are colocated with the wet spots of the soil surface. Further, regions or subareas consisting of several nearby measurement points with high normal velocities and the areas of the wet spots on the surface of the ground are visually highly correlated. The result of combining the high-velocity measurement points (red points) and the visual photo image of the scanned area is shown in Figure 7a. The red points were accumulated in the frequency band 420–650 Hz. It is clearly seen that locations of the wet spots and the high-velocity points are highly correlated.

[35] A simple model was suggested to explain this phenomenon (Figure 7b). Heavy equipment used in the construction of the road (graders and rollers) produced regions with different strain states due to inhomogeneity of the road material. Regions with higher strain states have greater densities and higher sound speeds [Lu et al., 2004]. These regions will have lower water permeability and lower porosity and dry more quickly after rain than regions with lower strain states, less density, and lower

Figure 6. Measured acoustic-to-seismic transfer function with frequency sub-band width equal to 30 Hz. (a) Transfer function over the blank spot ($f = 535$ Hz); (b) transfer function over antipersonnel mine ($f = 305$ Hz).
sound speeds. Numerical investigations show that a ground with low density and low sound speed has a higher magnitude of the TF. This explains the correlation between the regions with higher magnitude of the TF and the wet regions. One may also speculate that spatial locations with high particle velocity in different frequency subbands may be due to variation of the depth of the inhomogeneity of the layer within the wet regions. The thin layers will give high particle velocity response at higher frequencies than the thick layers.

[36] The spatial locations of the strongest response of the TF may be revealed in the space-frequency volume by drawing contours of different levels for the normalized TF. The normalized TF was obtained by dividing the velocity response for each frequency at each scan point by the maximum value of velocity on the scan area. Figure 8a shows these space-frequency contours of the normalized transfer function with the normalized transfer function level equal to 0.6. Owing to the fact that the normalized TF is used and the contour levels are fixed in Figure 8a, the top view of this function, shown in Figure 8b, is a little different from Figure 7a. However, Figure 8b shows that the spatial distribution of high-intensity points on the scanned area and the positions of these spatial points have a strong correlation with positions of the wet spots on the ground surface.

[37] It is reasonable to suppose that bright spots of different shapes often observed in the acoustic-to-seismic

Figure 7. Connection between the natural ground variability and the physical properties of layered ground. (a) Combined images of the ground over the blank spot and spatial distribution of scan points with high-velocity response in frequency range 420–650 Hz (red dots); (b) the model of the ground.

Figure 8. The normalized acoustic-to-seismic transfer function. (a) Contours in the space-frequency volume; (b) spatial distribution of higher-intensity points in the transfer function obtained by top view on this volume visualization.
TF over the selected spatial area and which cause high false alarm rates are result of the variability of the ground properties. Owing to these variations, different spots of the ground will provide different values of the TF. Additional inhomogeneity of layer thickness also will lead to the appearance to bright spots on the ground surface at the different frequencies. The one layer viscoelastic model of the ground is a very simple model; however, it explains the experimentally observed phenomena observed.

7. Summary

[38] In this paper the layered viscoelastic model of the ground was used to describe sound interaction with the ground. The influence of the natural variability of ground parameters on the acoustic-to-seismic transfer function was numerically investigated using the matrix technique. Effects connected with the finite size of the sound source were considered. It was shown that the finite size of the acoustical source in air may lead to widening of the maximum of the transfer function.

[39] The resonance approach was used for obtaining parameters of layered viscoelasticity of the ground through the outdoor measured TF. Using the resonance positions of extremes of the TF at the first stage of reconstruction permits us to decrease the number of estimated parameters of the media from 17 to 6. Final estimation of parameters for the layered elastic model of the ground was calculated within the set of parameters obtained by the resonance approach. Good agreement between the calculated and the measured transfer function was obtained.

[40] Analysis of dependencies of the TF from physical parameters for a model consisting of one elastic layer overlaying an elastic substrate was used to explain why the natural variations of ground parameters will cause greater fluctuations in the magnitude of TF at high frequencies than at low frequencies over the same spot of the ground.

[41] Analysis of outdoor measured frequency responses of a blank area for different frequency subbands shows spatial correlation between the high magnitude of the TF and spatial distribution of wet spots on the surface of the ground. A simple physical model based on numerical investigations of the effect of ground variability, which explains this phenomenon, was suggested.

[42] The model and data analysis confirm a strong influence of the layered ground parameters on the acoustic-to-seismic transfer function, which can increase false alarm rates for acoustic methods of land mine detection. The result of this paper may aid in understanding the nature of the variability of the acoustic-to-seismic TF and in establishing an improved acoustic prediction capability in natural ground environments.

[43] Acknowledgment. This work was funded by the Office of Naval Research under grant N00014-02-1-0878.

References


V. N. Fokin, M. S. Fokina, W. B. Howard, J. M. Sabatier, and N. Xiang, National Center for Physical Acoustics, University of Mississippi, 1 Coliseum Dr., University, MS 38677, USA. (vfok@olemiss.edu)