Validation of Bayesian design for broadband microslit panel absorbers using causal inference

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ABSTRACT:
This paper discusses experimental validations of multilayer microslit panels (MSPs) designed via Bayesian inference to obtain both high sound absorption and wide bandwidth simultaneously. Microslit perforation in thin panels is similar to microperforated panels [Xiang, Fackler, Hou, and Schmitt (2022). J. Acoust. Soc. Am. 151(5), 3094–3103]. MSP absorbers in single-layer configurations are functioning in a limited frequency range. By stacking the MSPs in multiple layered structures, absorbing performance may be widened in frequency ranges while retaining high absorption coefficients. Besides design challenges of multiple MSPs in layered structures to fulfill a practical requirement and minimize fabrication complexity, this paper further discusses challenges in experimental validations when experimental results undesirably deviate from the initial Bayesian design. Causation analysis is applied to the validation efforts where a causal model-based inference effectively provides causal reasoning of fabrication inaccuracies when experimental results undesirably deviate from the initial Bayesian design. Causation analysis is applied to the iterative validation process. © 2023 Acoustical Society of America.

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I. INTRODUCTION

This work focuses on a validation effort for the Bayesian design of multilayered microslit panel (MSP) absorbers applying causal inference, besides the design of broadband sound absorbers using multiple sets of MSP absorbers. Microslit perforations in thin panels are assessed for their potential as sound absorbers in wide bandwidths when designed in layered structures. Since the 1990s, microperforation theory has led to a boom in innovative sound absorbers (Maa, 1975). Similar to the MPP sound absorbers, a single set of the MSP absorbers is effective when experimental results undesirably deviate from the initial Bayesian design. Causation analysis is applied to the iterative validation process, where a Bayesian network is backed by an air cavity before a rigid termination. However, small variances in the impedance of each individual panel cascade into larger changes in absorption performance in the overall assembly. This paper focuses on the challenging design task of how to experimentally validate the multiple MSP layers with a parsimonious number of layers. This paper presents further development from the MPP design (Xiang et al., 2022) by applying the causation analysis in the iterative validation process, where a Bayesian network is applied for the causal reasoning during the iterative validations.

Multiple-layer configurations are a direct approach to widening the absorbing bandwidth while retaining the high absorption. However, each additional MSP layer leads to an increased number of MSP parameters (Bravo et al., 2013; Miasa and Okuma, 2007; Ruiz et al., 2011), which in turn multiply challenges when creating such a multilayered

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The MSP absorbers are made of thin panels microslit in submillimeter width (Maa, 2001). Similar to the MPP sound absorbers, a single set of the MSP absorbers is effective within limited frequency ranges when the absorption coefficient is high (Maa, 1998; Xiang et al., 2022).

This paper applies the causal inference to the validation effort when designing multiple MSPs in layers. This causal inferential validation discussed here was not discussed previously in the most recent MPP design (Xiang et al., 2022). In the multilayered configuration, only the very end layer is backed by an air cavity before a rigid termination. However, small variances in the impedance of each individual panel cascade into larger changes in absorption performance in the overall assembly. This paper focuses on the challenging design task of how to experimentally validate the multiple MSP layers with a parsimonious number of layers. This paper presents further development from the MPP design (Xiang et al., 2022) by applying the causation analysis in the iterative validation process, where a Bayesian network is applied for the causal reasoning during the iterative validations.

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absorber (Carbajo et al., 2020; Qian et al., 2014). Similar to a design procedure of broadband multilayered MPP absorbers (Xiang et al., 2022), the design of multilayered MSP absorbers starts from the practical requirement of sound absorption.

First, this paper introduces the MSP prediction model (Maa, 2001) used to predict the performance of multilayer MSP absorbers. This work closely follows the recent publication (Xiang et al., 2022) about a Bayesian design process for MPP absorbers. This paper reports on three distinct advances: The design process applies a clearly different microslit model along with a more efficient iterative approach to multilayering than that of Xiang et al. (2022). Another important distinction is that this paper for the first time formulates the multilayer MSP predictions in the form of a causal model used in the Bayesian network for the causal inference. Moreover, this paper sheds light on details of an exploratory sampling method for the Bayesian inferential design, which goes significantly beyond the recent publication (Xiang et al., 2022). Last, this work further applies a causal inference approach for the validation. The validation process for the broadband MSP absorbers using the causation analysis has not been reported in the acoustical literature, particularly when applied in the practical design and validation of multilayered MSP absorbers within the Bayesian framework.

In the rest of the paper, Sec. II formulates a parametric model for N-layered MSP absorbers. Different from the previous paper, this model formulation is of central importance for both the Bayesian design and the causal inference of the experimental validation. Section III briefly discusses Bayesian analysis of two inferential levels for the design of the MSP absorbers in multilayers, followed by the mathematical foundation of a Bayesian Markov chain Monte Carlo sampling for an efficient exploration of high-dimensional parameter spaces. Section IV provides a brief Bayesian design process. Section V discusses the causal inference applied in experimental validations, and finally, Sec. VI discusses a number of challenging issues before the paper concludes in Sec. VII.

II. PARAMETRIC MODEL FORMULATION

Maa (2001) pioneered the microslit theory for predicting MSP absorbers. The microslitting of single panels is of central importance for multiple layer configurations. An iterative nesting of the panel surface impedance is described to cascade multiple MSP layers with an air cavity behind each MSP layer, detailing a numerical implementation of the transfer matrix method (Xiang et al., 2022).

A. Predictive model of MSPs

Maa (2001) derived a prediction model for MSPs as an extension of his pioneering work on the microperforation theory (Maa, 1975). A microslit is considered as a large number of “thin tubes” side-by-side connected together along a line, forming a microperforated slit. When sound waves impinge upon a MSP with microslits of submillimeter widths, the sound wave forces air particles to move back-and-forth inside the microslits. This movement of air particles inside microslits causes friction losses due to the boundary-effect, inherently creating significant acoustic resistance. Maa (2001) describes the acoustic impedance on the surface of MSPs facing normal incident sound waves as

\[ Z_{\text{MSP}} = R_{\text{MSP}} + j \omega m_{\text{MSP}}, \]  

with

\[ R_{\text{MSP}} = \frac{3 \eta t}{q \rho c r^3} \left( \sqrt{1 + \frac{r^2}{18}} + \frac{\sqrt{2} \sqrt{r}}{6} \right) \]  

and

\[ m_{\text{MSP}} = \frac{t}{q c r} \left[ 1 + \frac{1}{\sqrt{5^2 + 2r^2}} + \frac{F(e) r}{t} \right], \]

where \( t \) is panel thickness, \( r \) is microslit half-width (equivalent to micropore radius), \( q \) is perforation rate, \( c \) is sound speed, \( j = \sqrt{-1} \), \( \omega \) is the angular frequency, and

\[ \chi = r \sqrt{\frac{\omega \rho}{\eta}} \]

represents a perforation coefficient. The air filling the microslits and surrounding the panel exhibits dynamic viscosity \( \eta \) and density \( \rho \). Equation (3) also contains a function of elliptical integral (Maa, 2001)

\[ F(e) = \frac{\pi}{2} \left[ 1 + \left( \frac{3}{2} \right)^2 e^2 + \left( \frac{1 \cdot 3}{2 \cdot 4} \right)^2 e^4 + \left( \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \right)^2 e^6 + \cdots \right], \]

with

\[ e = \sqrt{1 - \left( \frac{l}{r} \right)^2} \],

where \( l \) is microslit half-length, leading to a microslit cross section area \( 4l/r \). The cross section area is decisive for the perforation rate \( q \) with the half-length implicitly in \( q \).

B. Iterative multilayer configuration

Figure 1 illustrates a multilayered MSP configuration. The normal incidence surface impedance \( Z_{c,n} \) right in front of the \( n \)th panel consists of both the panel impedance \( Z_{\text{MSP},n} \) in Eq. (1) and the cavity impedance \( Z_{\text{cav},n} \) right behind it,

\[ Z_{c,n} = Z_{\text{MSP},n} + Z_{\text{cav},n}, \quad n = 1, 2, \ldots, N. \]

The \( n \)th air cavity of depth \( d_n \) in front of a termination impedance \( Z_{c,n-1} \) can be determined (Xiang and Blauert, 2021) by a transfer matrix operation.
The surface impedance but Eqs. (7)–(9) facilitate an iterative numerical implementation since the surface impedance with

\[ Z_{\text{MSP}}(\theta) = \text{Im} \left( \sum_{n=1}^{N} \frac{Z_{n}}{\rho c} \right) \]

Equation (10) represents the parametric model of the absorption coefficient. The parametric model of an N-layered MSP absorber is generally defined over a 4 × N-dimensional parameter space. The following derivations use \( M(\theta) \) to denote this parametric model. The model contains an array of MSP parameters \( \theta = [t, r, q, d] \). All parameters denoted by bold letters represent vectors encapsulating parameter values of N individual layers; for example, \( r = [r_1, r_2, ..., r_N] \) represents N microslit half-widths.

### C. Causal model of MSP absorbers

A causal model is a graphical diagram consisting of vertices (circles, oval, or boxes) and directed links that represent the functional relationships between influencing variables (Pearl, 2009). The causal model established for this application essentially represents a graphical illustration of the parametric MSP model as derived in Eq. (10) through Eqs. (1)–(9). Figure 2 illustrates this causal model established for the MSP prediction, design, and validations. It provides the designer with a convenient way to comprehend and develop prediction algorithms. It also provides a causal structure for the designer to pursue a causal analysis via experimental observations when the ultimate experimental results show any deviations from the design requirement (see Sec. VB).

In Fig. 2(a), vertex \( Z_{\text{MSP}} \) is a collider-type element; three “parent” variables have causal influences on the MSP panel impedance \( Z_{\text{MSP}} \) as mathematically expressed by Eq. (1) through Eqs. (2)–(6), while vertex \( Z_{\text{cav}} \) has only one single parent variable influencing it. Another “collider”-type vertex \( + \) represents the total acoustic impedance of the nth single MSP layer; both vertices \( Z_{\text{MSP}} \) and \( Z_{\text{cav}} \) are its parents. The total acoustic impedance is considered as a higher-level vertex labeled by the “nth single layer” framed inside a dashed-line box as expressed in Eq. (7). To be more precise, each individual MSP layer is dictated by three MSP parameters \([t_n, r_n, q_n] \) along with a back air cavity of depth \( d_n \). This causal structure is graphically illustrated by considering the dashed-line region of Fig. 2(a) as a black box, containing hidden vertices for the lower-level modeling steps. When each single MSP is rigidly terminated with an air gap, it can also be converted (expressed) in the form of an absorption coefficient as expressed by a “sink”-type vertex, \( x_n \).
Furthermore, when abstracted to be the higher-level vertices containing the hidden vertices, it clarifies the causal structure that \(N\) individual such impedances are cascaded via the impedance vertex in Fig. 2(b); namely, the “forked” links direct each layer impedance from Fig. 2(a) to “chained” vertices, expressed through an iterative algorithm as in Eq. (7) through Eqs. (8) and (9). They result in an overall surface impedance of an \(N\)-layered MSP absorber when rigidly backed at the impedance vertex \(Z_1\) as illustrated in Fig. 2(b). The chained impedances lead to the sink vertex, resulting in an overall absorption coefficient \(\gamma_N\) as expressed by Eq. (10); it yields the absorption performance as will be designed by applying the two-leveled Bayesian inference as discussed later.

### III. UNIFIED DESIGN FRAMEWORK USING A BAYESIAN ANALYSIS

Bayesian model selection and parameter estimation when applied to MPP absorbers were detailed in a previous publication (Xiang et al., 2022). Therefore, this section only briefly summarizes the design process of tentatively \(N\)-layers of MSP absorbers, so that this paper can focus on more details of practical implementations and an experimental validation effort in Secs. IV C and V, respectively. The design faces twofold challenges, especially when the practical treatments call for a high absorption in a wide bandwidth of frequency. An increase of the number of layers would be one possible way to meet both the broad frequency range and the high absorption. As already discussed in Sec. II, four MSP parameters dictate the absorbing effect of one single MSP layer. First, the layer number increasing elaborates the manufacturing complexity and stands in need of increased absorber thickness. Second, each additional layer will inevitably increase four extra MSP parameters. Therefore, acoustical practice highly prefers a concise MSP configuration for the designed performance.

#### A. Design data for Bayesian analysis

This work designs the sound absorber from a practical application requiring an absorption being not below 0.8 between 500 Hz and 2.75 kHz [differing from the one required in Fig. 3 of the recent work by Xiang et al. (2022)]. The application does not raise any specific requirement outside this frequency range. Note that the absorption coefficient is defined as the ratio of power flux incident to the plane of the absorber to the power flux reflected back to the absorber, and its unit is expressed in either “bans” or “decibans” in honor of Thomas Bayes (Jeffreys, 1961).

#### IV. IMPLEMENTATION OF BAYESIAN DESIGN

This section details implementation issues related to both inferential levels. The model selection is dedicated to estimating a parsimonious number of MSP layers. The parameter estimation is conducted for estimating the MSP parameters once the number of layers is determined via the model selection. Clearly extended from the most recent publication (Xiang et al., 2022), this section describes an efficient approach using nested sampling (Jasa and Xiang, 2012; Skilling, 2004) in detail to benefit interested readers for their own exploration work. This approach applied to multilayer broadband MPP/MSP absorbers has not yet been fully documented in the published literature.
In the current design, a sound absorber is made of multiple MSP sets. It fulfills the design requirement of absorption scheme. The design scheme \( \mathcal{D} \) essentially provides an upper bound \( B_u(f_k) \) and a lower bound \( B_l(f_k) \) at any frequency \( f_k \) of interest. There are \( K \) discrete frequencies within the designed frequency range. At each frequency, the model predicts the absorption coefficient \( \kappa(f_k) \) for multilayer MSPs as discussed in Sec. II.

The parameter vector \( \theta \) encapsulates four types of MSP parameters: microslit half-width \( t_m \), perforation rate \( q \), panel thickness \( t_n \), and air cavity depth \( d_n \) with \( n = 1, 2, \ldots, N \) for an \( N \)-layer MSP configuration. The prediction model \( M(\theta) \) in Eq. (10) encapsulates these parameters.

### A. Likelihood assignment

The likelihood expresses the probability of errors between the desired sound absorption \( \text{(Xiang, 2020)} \) within the design scheme and the absorption coefficient \( \kappa(f_k) \) determined via the parametric model of multiple MSP layers. With these errors being finite, yet unknown values, the principle of maximum entropy (MaxEnt) is applied to the probability of errors at each frequency with the only information \( I \) resulting in a Gaussian probability density function. Note that the assigned Gaussian function is distinctly different from assuming the errors to be Gaussian \( \text{(Xiang et al., 2022)} \). It represents the consequence of the MaxEnt with only available information \( I \) on the error function. Furthermore, with no knowledge on dependence between the errors, an independence is assigned among the errors at each frequency. This independence represents the effective measure to prevent reducing the entropy of the error probabilities \( \text{(Jaynes, 1968)} \). The overall likelihood function based on the product rule \( \text{Xiang, 2020} \) becomes

\[
L(\theta) = p(D|\theta, M_N, I) = \prod_{k=1}^{K} \frac{1}{\sigma_k \sqrt{2\pi}} \exp \left( -\frac{E_k^2}{2\sigma_k^2} \right),
\]

where \( E_k \) represents the error at each frequency \( f_k \) \( \text{Xiang et al., 2022} \).

\[
E_k = \begin{cases} 
\kappa(f_k) - B_u(f_k) & \text{for } \kappa(f_k) > B_u(f_k), \\
B_l(f_k) - \kappa(f_k) & \text{for } \kappa(f_k) < B_l(f_k), \\
0 & \text{otherwise}.
\end{cases}
\]

Quantity \( \sigma_k \) is a hyperparameter representing the standard deviation of the error. Similar to the recent work \( \text{Xiang et al., 2022} \), the designer assigns \( \sigma_k \) a value of 0.05 between 500 Hz and 2.75 kHz. Otherwise, a high value of 0.5 is assigned outside the above range.

### B. Prior probability assignment

Before involving the design scheme \( \mathcal{D} \), one of the two inputs of the Bayes’ theorem, the assignment of the prior \( \Pi(\theta) \) in Eq. (12) is pursued, based merely on available knowledge. Maa’s theory \( \text{Maa, 2001} \) provides the only available knowledge that the microslit widths are on the order of submillimeters, while the panel thickness is limited to the availability of material panels. Without further knowledge, except only that the probability density must be integrated to unity over the entire parameter range \( \text{Xiang, 2020} \), the MaxEnt comes once again into the prior assignment. The MaxEnt assigns probability densities to be uniformly distributed over wide parameter ranges \( \text{Jaynes, 1968} \).

The parameters for each MSP layer are assigned to uniform distributions as follows:

- \( \Pi(r) = \text{Uniform (0.1 mm, 0.5 mm)} \) —for microslit half-width;
- \( \Pi(q) = \text{Uniform (0%, 20%)} \) —for perforation rate;
- \( \Pi(d) = \text{Uniform (0.5 cm, 5 cm)} \) —for MSP cavity depth.

In the following design example, a fixed panel thickness of 2.56 mm turns out to be reasonable. The fixed panel thickness \( t_n \) is also indicated by a dashed-line vertex in Fig. 2(a). This reduces the number of MSP parameters to \( 3 \times N \).

### C. Estimation of number of layers

This section exposes details for the major procedure in selecting the number of MSP layers on an implementation level, which has not yet been sufficiently described in acoustical applications for ease of implementation within the recently published literature, including Landschoot and Xiang \( \text{(2019)} \) and Xiang et al. \( \text{(2022)} \). However, a theoretical exposition was given in acoustical literature, for example, by Jasa and Xiang \( \text{(2012)} \).

The Bayesian evidence in Eq. (12) is approximated by summing up a discrete likelihood sequence \( \text{Jasa and Xiang, 2012} \)

\[
Z = \int_\theta L(\theta) \Pi(\theta) d\theta = \int_\mu L(\theta) d\mu \approx \sum_{q=1}^{Q} L_q \Delta \mu_q,
\]

where \( d\mu = \Pi(\theta) d\theta \), and the variable \( \mu \) is termed prior mass. Skilling \( \text{(2004)} \) approximated the infinitesimal prior mass \( d\mu \) using

\[
\Delta \mu_q \approx \mu_{q-1} - \mu_q \text{ with } \mu_q \approx e^{-q/P},
\]

where \( P \) is the number of an initial population as elaborated in the following. The nested sampling as formulated in Eq. (16) on the right-hand side is to explore the likelihood function over the entire prior mass \( \mu \) to find an increasing likelihood sequence

\[
0 < L_1 < L_2 < \cdots < L_{Q-1} < L_Q = L_{\text{max}},
\]

which corresponds to a decreasing sequence of differential prior mass

\[
l = \mu_0 > \mu_1 > \mu_2 > \cdots > \mu_{Q-1} > \mu_Q \approx 0.
\]
The nested sampling begins to explore the entire MSP parameter space (Jasa and Xiang, 2012; Skilling, 2004) via a prior mass, given that the prior assignment in Sec. IV A is wide enough over the prior distributions. For each model, the implementation in this work initially involves \( P(\mu = 50) \) uniformly distributed samples at the start of the nested sampling [see Eq. (17) in Sec. IV A]. Figure 3 illustrates the conceptual implementation using an initial population of five instead of 50 for illustrative purposes. Throughout the sampling process, these 50 samples as a fixed population remain active, while the sample associated with the lowest likelihood [labeled by \( L_1 \) in Fig. 3(a) and by “1” in Fig. 3(b)] is pushed outside consideration but saved into a growing list as illustrated in Figs. 3(c) and 3(d).

During the sampling, MSP parameters are perturbed repetitively until the perturbation makes a likelihood value higher than the lowest one from the previous iteration. Upon fulfilling this likelihood constraint, the perturbed sample replaces the discarded one in the active population, keeping it as a fixed population size, that ends the current sampling step (Ashton et al., 2022). Iterative sampling is stopped when an increment of likelihood between subsequent samples falls below \( 10^{-7} \) bans.

Figure 4 illustrates one example of the likelihood sequence sampled using the implemented nested sampling. For a three-layer model as an example, the nested sampling explores the entire MSP parameter space by spending 1085 iterations to converge. The nested sampling is pursued in its random iterations. To make sure that the nested sampling converges, the estimated evidence values for each layer (from two to five layers) are independently collected over 16 runs. A comparison of the averaged value in this way with individual runs indicates sufficiently small variations to show a sufficient convergence for reliable estimations. Table I lists the Bayesian evidence and the Bayes factors \( f_{n-1} \) for \( n = 2, 3, 4, 5 \), estimated from averaged Bayesian evidence. The estimated Bayes factor \( f_{3,2} \) arrives at the highest value, indicating that this design prefers a three-layer MSP configuration.

The Bayesian model selection relies on evidence estimations that require much computational load in estimating the Bayesian evidence values for a handful of models. In this reported design effort, Bayesian evidence \( Z_N \) is selected among \( M_N(\theta) \) for \( N = 1, \ldots, 5 \).

### D. Estimation of design parameters

The parameter estimation details the three-layer design in this example. Table II lists the parameter values of the maximum a posteriori (MAP) estimates. Figure 5 shows the design absorption curves with these parameters. The original design scheme as denoted by \( D \) (in Secs. III A, III B, IV A, and IV B) is also plotted for ease of comparison. The single-layered absorption coefficient curves are also shown in Fig. 5. The three individual resonance curves of the absorption coefficients are graphical representations of the sink-type vertices \( \zeta_n \) with \( n = 1, 2, 3 \) in the causal model illustrated in Fig. 6.

<table>
<thead>
<tr>
<th>Number of layers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evidence (decibans)</td>
<td>300.5</td>
<td>350</td>
<td>475</td>
<td>472</td>
<td>470</td>
</tr>
<tr>
<td>Bayes factor (decibans)</td>
<td>49.5</td>
<td>125</td>
<td>–3</td>
<td>–2</td>
<td></td>
</tr>
</tbody>
</table>

TABLE I. Bayesian evidence and Bayes factors. They are measured in the unit “decibans,” indicating tens of logarithmic operations of respective quantities (Jeffreys, 1961).
Fig. 2(a). Note that only stacking three individual layer panels in the correct order will yield the designed absorption performance, not directly averaging them.

V. CAUSAL INFERENCE IN VALIDATIONS

To validate the MSP absorber design, multiple sets of three-layer MSP panels are created using an automated laser cutter. Their parameters are listed in Table II. The laser cutter may introduce small fabrication inaccuracies when cutting the microslits. This experimental validation employs polycarbonate clear sheets; their slit shapes perpendicular to the panel surface are relatively insensitive to laser cutting. Nevertheless, the microslits may feature deviated shapes from the model pioneered by Maa (2001) during manufacturing, as also noted by Aulitto et al. (2021) and Zielinski et al. (2019). If the experimental results conform to the design scheme, the experimental effort will validate the multilayered design. However, the fabrication inaccuracies may cause unacceptable deviations from the design scheme, and the multilayered configuration may pose further challenges in determining the causes of the unacceptable results. Instead of accurately determining the deformed shapes in microslits, this work effectively estimates the individual microslit parameters with slightly deviated values due to laser cutting through causal inference via a Bayesian network. This section further describes the analysis method applied to this validation effort.

A. Experimental setup

The two-microphone transfer-function method (Chen et al., 2022; Chung and Blaser, 1980) is used for experimental measurements of the absorption coefficients among multiple groups of MSP samples in an impedance tube. Figure 6 shows photographs of one set of the three-layer MSPs, the air gap keepers, and the impedance tube setup. The tube has a diameter of 5.2 cm, which is suitable for normal incident absorption measurements up to 5 kHz. Figure 6(c) shows how the sample panels are mounted on the tube end. Figure 7(a) illustrates one group of initial experimental results obtained using the tube measurement, when putting the three layers together as designed. Critical parameters relevant to the tube measurements were estimated prior to the experimental measurements of the samples (Chen et al., 2022). However, these specific experimental results could not meet the design scheme over the frequency range between 500 Hz and 2.75 kHz as required by the application. In this multilayered configuration, it is extremely challenging to determine which layer or layers differ from the initial design and prevent the absorption performance from falling into the design scheme. Therefore, further analysis tools are necessary.

B. Causal inference for experimental deviations

For a sufficient understanding of why the initial experimental results in Fig. 7(a) undesirably deviate from the design scheme, this work has further conducted causation analysis in an iterative manner during further validation steps (Hoeft et al., 2021). For this causation analysis, the causal model in Fig. 2 as introduced in Sec. II C plays a crucial role (Pearl et al., 2019). The design target is expressed by the “sink” vertex $a_N$. The causal structure represented by the causal model conveniently helps the designer trace
possible causes inversely from the sink vertex $x_N$ to the influencing variables (vertices). In causality literature (Pearl et al., 2019), they are considered as vertices in “parent,” “grandparent,” or even “great grandparent” generations. With the tube measurement data at hand, this causation analysis represents how the designer incorporates the experimental data in the learning process to reveal what causes the observed deviations.

C. Bayesian network for causal reasoning

Once again, the present work pursues the causation analysis within the Bayesian inferential framework. The analysis within the current Bayesian framework represents exactly an inferential process, namely, the Bayesian causal inference. Each time when unacceptable deviations between the absorption coefficient of three-layered MSP absorbers in comparison with the design prediction are observed, the single-layered absorption coefficients from the experimental measurements are used to estimate the actual MSP parameters for the fabricated panels to infer how much they deviate from the original design as listed in Table II. This causal inference is once again carried out using the model-based Bayesian parameter estimation, also known as estimation using the Bayesian network.

The causal model indicates that the overall absorption coefficient is dictated by three MSP layers. Inaccuracies of the fabrication process are intuitively hypothesized for the observed deviations. Tracing inversely from the sink vertex $x_N$ (with $N = 3$), acoustic impedances $Z_1$, $Z_2$, and $Z_3$ of each single MSP layer are governed by the four MSP parameters as illustrated in Fig. 2(a). Inaccuracies due to the back cavities are straightforwardly examined and are ruled out immediately. What remains at this point are only three MSP parameters; among them, the panel thickness $t_n$ is fixed during the design as indicated by the dotted-line vertex and can be taken out of consideration. The focus on possible inaccuracies is logically placed on the individual MSP parameters $r_n$, $q_m$. Their resulting values by the laser cutter due to possible inaccuracies need to be evaluated.

For this purpose, the “network” for the individual single layers in Fig. 2(a) serves as the relevant model for further causation analysis. If every single panel was terminated by a rigid backing, it could result in single-peaked resonant absorption behaviors expressed by the “sink”-type vertices $x_n$ ($n = 1, 2, 3$). Figure 5 also illustrates these individual single-layer curves (dotted lines) of the absorption coefficients from the initial design. To estimate their resulting inaccuracies of actual MSP parameters due to the laser cutting, each individual single layer with its respective air cavity terminated by a rigid backing is measured in the impedance tube. The experimental data obtained this way are used to estimate the resulting MSP parameters fabricated by the laser cutter.
To this end, the estimation based on the Bayesian network model in Fig. 2(a), as formulated in Eq. (11), is once again applied to the parameter estimation. This parameter estimation specifically engages the single-layer model $M_1(\theta)$ with fixed known values of $d_0 = 2.56$ mm and $d_n$ (as listed in Table II). Figure 7 also illustrates the tube measurement results along with the estimated curves of absorption coefficients via the Bayesian network. Each rigidly backed single-layered MSP absorber in Figs. 7(b)–7(d) demonstrates a single-peaked resonant absorption behavior over a rather narrow frequency range, and they are located differently from the single-layered prediction determined by initially designed parameters. These experimentally measured absorption curves also exhibit even narrower peaks around 2.5–2.8 kHz. These narrow peaks are identified as vibrational resonances of each single panel, although the three-layered configuration makes them less noticeable in Fig. 7(a). Recent works by other authors have also reported the panel vibrational resonances in a single-layered MPP or MSP absorbers (Falsafi and Ohadi, 2017; Zielinski et al., 2019).

Table III also lists the estimated values in comparison with the initially designed ones. These estimated results clearly reveal the cause-effect relationship, supporting reasoning of the observed deviations. Figures 7(c) and 7(d) particularly indicate that these two individual panels were fabricated with relatively larger deviations than those of layer 1 as shown in Fig. 7(b). The listed differences in Table III also help instruct the laser cutter to correct the previous fabrication. In addition to many groups of panels prepared to attain the desired results, the above-mentioned corrections can be divided into two primary ways. On the one hand, specific designs of MSP panels were modified to compensate for the errors based on the Bayesian estimation results. For instance, if the measured slit width was smaller than the desired one in Table II, the slit widths of the panels were proportionally modified to achieve better performance. On the other hand, changes were made in manufacturing. Instead of using a slow, high-power laser to cut panels in one run, the laser cutter is adjusted during the new manufacturing process to go faster with lower power, yet with multiple steps of cutting. This would ensure that the slits are as clean as possible.

Based upon this causation analysis using the causal model-based Bayesian network, the created samples during the following fabrication iterations lead to the resulting absorption coefficient as shown in Fig. 8. Three distinct ripples are clearly recognizable within the absorption range between 0.8 and 1.0, which are closer to the initial design than the first experimental result in Fig. 7(a). This experimental result demonstrates a desired absorption performance that largely meets the design scheme, sufficiently validating the initial Bayesian design, although there are still deviations slightly outside the design scheme. Further iterations of laser cutter corrections could be applied to guide the laser cutter for even closer results if desirable.

VI. DISCUSSION

Manufacturing of designed MSPs is inevitably subject to wide variation and inaccuracies. The validation process discussed above reveals that the MSPs will exhibit much less inaccuracy than that of MPPs evidenced through the experimental results during the validations as presented in a recent publication (Xiang et al., 2022). For a three-layer design of the MPP absorber [see Fig. 8 in Xiang et al. (2022)], the experimental results exhibit absorption values largely toward the upper bound of the design scheme, with much smaller (three) ripples. This is obviously because the hundreds and thousands of micropores are required by a given perforation rate, far more than the MSPs. Every single micropore may be subject to inaccuracies during the fabrication process, which may lead to larger accumulated inaccuracies. Due to submillimeter micropores, it will be extremely challenging to control the fabrication inaccuracies of the MPP manufacture, while it is relatively less challenging when manufacturing the MSP panels. Furthermore, the MSP panels will yield more unobstructed perforations for a given perforation rate when transparent/translucent absorbers [as in Figs. 6(a) and 6(b)] are needed.

If the design scheme would require different absorption band features, the Bayesian inferential designs discussed in this paper would still be suitable. This section discusses two different examples. Figure 9(a) illustrates a design scheme

![Fig. 8. (Color online) Sound absorption coefficients (solid/dashed lines) of a three-layer microslit panel absorber, experimental validation result through iterative corrections upon the causal inference of fabrication errors. The design scheme and three-layer design of absorption are also presented for ease of comparison.](image-url)
with the absorption band between 700 Hz and 2 kHz requiring the lower design bound for the absorption coefficient to be as high as 0.95, while higher than 2.3 kHz, the absorption is gradually reduced from between 0.2 and 0.6 down to between 0.06 and 0.3. Nested sampling absorption designs based on the three-layer model are also illustrated in grayscale curves. The grayscale from light to dark graphically presents a gradual increment of the posterior values of each random design. For such a stringent requirement, it is highly demanding for the nested sampling to converge to the parameter values of each random design. For such a stringent requirement, it is highly demanding for the nested sampling to converge to the parameter values of each random design. The converged design tightly fulfills such a demanding design scheme. In Fig. 9(b), absorption coefficients ranging from 0.8 to 1.0 are required, similar to the one in Fig. 5. However, the absorption band needs to be much broader, ranging from 1 to 8 kHz. Only a five-layered model would show satisfactory fulfillment of such a broadband design scheme.

In practice, it will only make sense to expand the number of layers or to tighten the absorption range [such as in Fig. 9(a) from 0.95 to 1.0] given that the real applications will need to meet such a stringent requirement. In architectural acoustics applications, this is often not the case.

VII. SUMMARY

This paper discusses experimental validations of multilayered MSP absorbers designed within the Bayesian framework. The design is pursued in multilayer configurations to widen the frequency range of required absorbing performance. The desired absorption performance comes from application requirements when a multilayer MSP absorber is under consideration. MSPs in multiple layers pose twofold challenges. First, the design with $4 \times N$ parameters of MSPs in general, $3 \times N$ for this work in particular, exhibits extremely high dimensionality of the MSP parameter space for the designer to determine both the parsimonious number of layers and a large number of MSP parameters. Second, it is challenging to reveal the cause and effect relationship between the initial design and actual samples used for experimental validations, when the experimental results show unacceptable deviations from the initial design.

A model-based design method using Bayesian inference is efficient to estimate both a parsimonious number of MSP layers and the MSP parameters for multilayered configurations. These MSP parameters are used to manufacture MSP absorber sets given the desired absorption performance. This paper has proposed and validated a parametric model for MSP sound absorbers that incorporates potential multilayered MSP absorbers with an iterative algorithm benefiting the numerical implementation of the transfer matrix method. Two inferential levels are involved for the model-based Bayesian design. The model selection level was first engaged in determining the parsimonious number of MSP layers that implement Occam’s razor to arrive at the parsimonious number of MSP layers.

Effort is also made to describe in detail the nested sampling for an efficient estimation of the key probabilistic quantity for selecting a minimum number of layers.

Normal incident tube measurements support experimental validations of the designed absorber required from practice over frequency ranges between 500 Hz and 2.75 kHz. During the validation effort, unacceptable deviations of the overall absorption performance of the samples created by a laser cutter motivate the further application of the causation analysis during an iterative validation process. Within the Bayesian inferential framework, the causality due to its mathematized formulation is also analyzed based on a causal structural model to infer the causal information. This efficiently benefits a practical design and its validation of broadband sound absorbers.

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