Instantaneous and time-averaged intensity flow measurements in scale models

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Abstract: Current methods of representing energy flows in rooms have been limited to stationary pressure contour maps or intensity vectors derived from finite-element modeling (FEM) software. Despite recent advances in FEM analysis, such models are at best approximations of real-world phenomena because of the assumptions made in obtaining these results. This paper presents an energy flow visualization tool that combines the well-accepted practice of scale modeling in performance venue acoustics design and recent advances in sensor technology to provide rapid and accurate real-time maps of intensity flows derived from simultaneously-measured pressure and velocity quantities in the studied space.

1. Introduction

For a long time, the study of sound energy propagation in architectural acoustics has been primarily restricted to measurements of pressure distributions at locations of interest. After all, much headway has been made in understanding how sound pressure correlates with auditory perception, and to a certain extent, sound intensity by virtue of its linear relation to the square of pressure. However, the relegation of sound intensity to that of a scalar quantity diminishes its usefulness as a vector quantity in depicting the direction of energy flow within a space.

1.1 Timeline of intensity measurement techniques

The first documentation of sound intensity measurements could be attributed to Tichy and Elko. In this work, they introduced the concepts of active and reactive intensity and a method for estimating these vector quantities based on the limited equipment they had back then. The comparison of experimentally-measured intensities and model-predicted intensities as the number of sources and their respective positions in the room are varied is a particularly interesting approach that is adopted in this paper. Despite the relative simplicity of the abovementioned configurations, discrepancies between measured and modeled quantities were evident in certain cases. Among many factors, it was found that the location of sound sources and their interaction with each other and the boundaries affected the intensity distribution in the room. Waterhouse et al. built upon this work by defining the conditions of a vortex, a specific type of energy flow that can only be demonstrated with intensity vectors. Waterhouse then proceeded to establish the governing equations relating pressure, velocity, and intensity for rectangular room modes and the conditions when vortices would occur.

Intensity measurements were first made using two pressure sensors, which due to their large size and consequently spacing between them, resulted in approximation errors that limited their usefulness to low frequencies only. Extensions to this method to allow for higher dimension intensity measurements include the six microphone array (two for each dimension) and spherical array methods (acoustic holography). However, few of these caught on in practice, possibly because of the relative complexity of their implementation outside of a controlled laboratory environment. Latest advances in sensor technology have resulted in smaller and more accurately collocated pressure and velocity sensors like Microflown’s PU probe, which is able to accurately measure both pressure and three-dimensional velocity components at a point in space. More information on some of these techniques and their limitations can be found in works by Jacobsen and Fahy.
1.2 Applications of intensity measurements in industry

Recent interest in source localization and sound propagation exposed the limitations of scalar pressure distribution as a representation of energy flows and rekindled interest in intensity measurements. While beamforming is the conventional method for this application, it requires substantially more microphones to achieve higher directionality and a larger operating frequency range in the presence of extraneous sources. In contrast, with the latest technology, source localization using measured intensity only requires one pressure sensor and a maximum of three velocity sensors depending on the number of dimensions desired. Additionally, intensity measurements can be made in situ instead of anechoic chambers without losing much accuracy, making them a more viable alternative to pressure measurements for deriving sound power in noisy conditions. Such measurements have already been employed in automotive manufacturing and could very well be applied to the quantification of sound fields in architectural acoustics.

1.3 Intensity measurements in architectural acoustics

Studies of intensity in architectural acoustics have been few and far between over the past decade. Meissner showed steady-state intensity flows in coupled-room models including instances of vortices at select frequencies. Nolan and Davy proved by simulation that wall absorption affects the flow of active and reactive intensity in rectangular rooms. However, these studies on intensity flow in rooms all share a finite-element modeling (FEM) or computer simulation slant that are at best approximations of real-world phenomena. FEM modeling is often the best alternative when it comes to analyzing energy flows because the governing differential equations rarely have closed-form solutions. In summary, it works by combining approximated solutions of said equations over small differential components that make up the larger system to provide an overall solution.

The few major studies that did indeed consider experimentally-measured intensity did so as a means of delineating the limitations of Waterhouse’s random wave model below the Schroeder Frequency and that of the uniformly-diffuse sound field assumption in reverberation chambers. Even then, these studies were largely restricted to measurements of a highly localized region within a larger volume, which is insufficient for describing intensity flows across the entire volume of interest. Intensity flows have been shown to be an important contributing factor in the overall acoustics of small rooms, e.g., recording studios. However, there is currently little if any consideration of intensity in the design of these spaces in the architectural acoustics consulting industry. Consequently, a more generalized method that brings together the advantages of scale modeling and simple hardware implementation is needed, one that is easily translatable to real-world practical limitations.

Xiang et al. animated experimentally-measured pressure data in coupled rooms using the time-domain solution. To the authors’ best knowledge, this is the first time that the pressure distribution across the entire cross-section of a room has been comprehensively demonstrated using experimental data, of which this work is a continuation.

1.4 Method for representing instantaneous intensity, steady-state intensity, and pressure simultaneously in rooms

Current representations of energy flow in the literature typically revolve around single-frame intensity quiver plots or pressure contour maps, which are only reasonably accurate depictions of the steady-state case. However, the process of sound propagation from the onset of stimulus has yet to be reported. This can only be demonstrated with instantaneous time-dependent intensity, not steady-state (time-independent) intensity. While the concept of instantaneous intensity had already been introduced by Mann et al. and espoused by Jacobsen as early as the 1980s, it has often been overlooked for its steady-state counterpart. In fact, the seminal paper by Waterhouse et al. introducing the concept of the vortex stopped short of discussing instantaneous intensity despite having introduced the two components that comprise it—instantaneous pressure and velocity. This was also the case for his second paper discussing the vortex specific to a rectangular enclosure, a trend that perpetuates until today. To the authors’ best knowledge, the results and the method for obtaining them presented in this paper is a first for presenting instantaneous pressure and intensity data simultaneously in a room that captures the progression of their transient behavior and their mutual dependency over time. This experiment is conducted in an eighth-scale model to model energy flows in a full-size room. This use of scale models in architectural acoustics is not new and could be extended to the design of more sophisticated spaces where the direction of energy flow is of greater importance, e.g., mitigation of sound leakage between adjacent exhibit spaces in a museum—essentially a coupled-rooms design problem.

This paper is split into two parts. The first part is an extension of Waterhouse’s work, where the simple steady-state case of a rectangular room with a rational ratio of sides and solvable equations for pressure, velocity, and consequently, intensity distributions is discussed. While Waterhouse presented stationary plots of intensity and stream function, the animations in this paper visualize the simultaneous variation of pressure and intensity as a function of time. The second part applies the same governing equations for deriving active and reactive intensity in the preceding section to a more complicated case of a room where the ratio of sides is not rational and axial vortex modes are not expected. However, more complex tangential vortex modes may exist. For this case, both transient intensity flows and steady-state intensity flows derived from the same experimentally-measured room impulse responses are studied.
2. Methodology

The method used to visualize complex pressure and intensity data, henceforth referred to as the “frequency-domain solution” technique, is different from the “time-domain solution” that was used in the preceding work.13 For this method, log-sine sweep stimuli (2.1 s duration per sweep) are played via a loudspeaker located in the top left-hand corner of an eighth-scale rectangular room model of dimensions 79 cm by 61 cm (full-size equivalent measuring 632 cm by 488 cm). Simple spherical pressure wavefronts are not expected due to significant source-boundary interaction occurring where the two walls are conjoined, an example of a case that is harder to model but ideal for the purposes of this experimental study. The walls are quarter-inch thick medium-density fiberboard panels and are assumed for this study to have infinite impedance. Pressure and two-dimensional velocity room impulse responses are individually measured at each point of a 10-by-14 point (140 points total) measurement grid with a pointwise spacing of 5 cm in real size. This is accomplished by moving a pressure-velocity (PU) probe across the room using a customized two-dimensional automated translocation mechanism. Impulse responses are only measured after the probe comes to a complete pause at the respective points. A total of 131 072 data points are recorded for each point on the grid at a sampling frequency of 62.5 kHz. With 256 averages being taken per point and accounting for the travel time between points, the total scanning time amounted to approximately 30h.

Overlapping blocks of 256 points (sliding time window technique) with starting points spaced a millisecond apart are obtained from the pressure and two-dimensional velocity room impulse responses. A Fast Fourier Transform (FFT) (zero padding included for finer frequency resolution) is performed on these blocks which converts them to the complex frequency domain, and the bin value corresponding to the desired frequency is taken as the representative values for pressure and two-dimensional velocity respectively for that time frame. For the steady-state case, the room impulse responses are first convolved with white noise before the abovementioned steps are performed.

Measuring impulse responses from the same log-sine sweep stimulus at various points across the room preserves phase by accounting for the latency of sound arriving at the corresponding locations, allowing for the progression of energy flow in the room to be documented. The arrival time of the first wavefront at an arbitrary point in the room with reference to the start of the recording \( t_0 \) is defined in terms of stimulus onset time \( t_0 \), speed of sound in air \( c \), and the straight-line distance from source to the point \( d: t_a = t_0 + (d/c) \), where the time of arrival increases with distance from source to measurement location.

Finer resolution animations can be obtained by shortening the time step between successive FFT blocks. The animation speed can also be adjusted post-processing by changing the time step between each plotted frame.

3. Theoretical bases: Intensity in the steady-state ideal case

The complex inner product is a key mathematical concept underpinning the derivation of intensity from complex pressure and velocity, for which numerous definitions exist.12,18,19 It is common to use the complex conjugate of one of the elements to maintain positive definiteness of the resultant complex inner product, as is the case for this work.

The complex time-independent intensity vector \( I_{xy} \) for a single-source, single-frequency sound field has real and imaginary parts and is defined as the complex inner product in the room impulse response with velocity, being the complex conjugated variable in this case,

\[
I_{xy} = \frac{1}{2} \cdot p_{xy} \cdot \varepsilon_{xy}^* = I + iQ, \tag{1}
\]

where \( I = \text{real}(I_{xy}) \) is the active intensity and \( Q = \text{imag}(I_{xy}) \) is the reactive intensity. Note that pressure and velocity here are also time-independent or time-averaged variables.

For the simple steady-state case of a 2-by-2 axial vortex mode in a hard-walled square duct (vortex mode in a rectangular room whose ratio of dimensions is rational) with \( p_{xy,t} = p_0 e^{\sin kx - i \cos ky} e^{(\omega t - \phi)} \) and \( \varepsilon_{xy,t} = -k (\sin kx + i \sin ky) e^{(\omega t - \phi)} \),

\[
\text{real}(I_{xy,\text{duct}}) = \frac{1}{2} p_0 e^{\sin kx \cos ky + \cos kx \sin ky}. \tag{2}
\]

The right frame of Mm. 1 shows the real part of \( I_{xy} \) superimposed on time-dependent pressure \( P_{xy,t} \) for one cycle of the mode. Waterhouse1 presented the pressure distribution equation without showing how it varies with time. When animated, pressure is observed to vary evenly in a circular fashion from high to low within each vortex cell. The rotating wavefronts demarcate the boundary between high- and low-pressure zones, with the direction of rotation being opposite for laterally adjacent cells. \( I_{xy} \), however, does not change throughout the cycle because it is the time-averaged or time-independent intensity in a strictly rotational flow path. It does, however, exhibit a tendency to rotate because the curl of active intensity in a vortex is non-zero.

Mm. 1. Animation showing change in instantaneous intensity while time-averaged intensity remains constant in a square room with rigid boundaries as seen in Fig. 1.
Subsequently, instantaneous intensity is defined as
\[ I_{x,y,t} = \frac{1}{2} \text{real}(p_{x,y,t} \cdot v_{x,y,t}). \]  

The variables \( p_{x,y,t} \) and \( v_{x,y,t} \) are the instantaneous or time-dependent pressure and velocity, quantities measured by the PU probe. Mathematically, these quantities are the respective time-independent quantities multiplied by \( \exp(i(\mathcal{X} \cdot t)/A_0) \), the time-dependence factor,
\[ p_{x,y,t} = p_{x,y} \cdot \exp(i(\mathcal{X} \cdot t)/A_0), \]
\[ v_{x,y,t} = v_{x,y} \cdot \exp(i(\mathcal{X} \cdot t)/A_0). \]

The left and right frames of Mm. 1 show the same pressure distribution over time for one cycle. However, the left frame differs from the right frame in that the intensity vectors \( I_{x,y,t} \) instead of \( I_{x,y} \) vectors rotate with the wavefront as it completes one cycle in contrast to \( I_{x,y} \) vectors that remain stationary.

Equation (1) takes time-independent pressure and velocity as inputs in calculating time-independent intensity. However, by replacing time-independent pressure and velocity with time-dependent pressure and velocity in that equation and using the principle that reciprocals cancel out when multiplied, the resultant solution is still time-independent intensity.
\[ \frac{1}{2} p_{x,y,t} \cdot v_{x,y,t}^* = \frac{1}{2} \left[ p_{x,y} \cdot \exp(i(\mathcal{X} \cdot t)/A_0) \cdot (v_{x,y} \cdot i\exp(-i(\mathcal{X} \cdot t)/A_0))^* \right] = \frac{1}{2} p_{x,y} \cdot v_{x,y}^* = I_{x,y} = I + iQ. \]

This hypothesis is subsequently tested by simulation in the same square room. Indeed, intensity derived from the complex inner product of \( p_{x,y,t} \) and \( v_{x,y,t} \) is similar to the results derived from \( p_{x,y} \) and \( v_{x,y} \) in Mm. 1, i.e., it does not change, thus proving our hypothesis. Most importantly, this method allows for time-averaged intensity or time-independent intensity to be obtained from experimentally-derived instantaneous pressure and velocity when the pressure and velocity distributions are unknown.

Consequently, the active time-averaged intensity (or active intensity) can be derived as
\[ I = I_{\text{active}_{x,y}} = \text{real} \left( \frac{1}{2} \cdot p_{x,y,t} \cdot v_{x,y,t}^* \right), \]  

and the reactive time-averaged intensity (or reactive intensity) derived as
\[ Q = I_{\text{reactive}_{x,y}} = \text{imag} \left( \frac{1}{2} \cdot p_{x,y,t} \cdot v_{x,y,t}^* \right). \]

The active intensity is rotational and has non-zero curl while the reactive intensity is irrotational and has zero curl. Consequently, the active intensity is responsible for vortex phenomena.
4. Experimental results

According to the Nyquist Frequency theorem, the spatial domain must be sampled greater than twice the highest frequency to be studied. Considering a scale model measurement grid spacing of 5 cm and that frequency scales up \( n \) times in an \( n \)-th-scale model, the upper limit frequency for the full-size room is derived as

\[
f_{\text{upper limit}} = \frac{c}{2s \cdot n} = \frac{343}{2 \cdot \frac{0.05}{8}} = 428.75 \text{ Hz},
\]

where \( c \) is the speed of sound in air, \( n \) is the scale of the model and \( s \) is the grid spacing.

In the preceding section, the governing equations relating intensity, pressure, and velocity were first implemented in an idealized steady-state model with a rational ratio of room dimensions and known pressure and velocity distributions, i.e., simple axial vortex modes are expected. Pressure wavefronts deflect intensity vectors as they propagate, causing them to rotate and appear as vortices.

In this section, actual experimental data is then examined for this vortex phenomenon when the room geometry is more complex, i.e., the ratio of room dimensions is not a rational number. Solving the tangential vortex modes equation provides a ballpark frequency where vortical flows are expected in an idealized hard-termination case. The equation and its solution are shown as follows:

\[
f_{\text{modal}} = \frac{c}{\sqrt{\frac{l^2}{L^2} + \frac{m^2}{M^2}}},
\]

where \( c \) is the speed of sound in air, \( L \) is the length of the room, \( l \) is the modal number corresponding to the length, \( M \) is the width of the room, and \( m \) is the modal number corresponding to the width.

This equation has two sets of solutions of two mode numbers each, namely, \([l, m] = [3, 1]\) and \([l, m] = [2, 2]\) that coincide at \( f_{\text{modal}} = 89\text{ Hz} \) for a full-scale room of dimensions \( L = 6.32 \text{ m} \) and \( M = 4.88 \text{ m} \).

Starting with the transient case, the four frames in Mm. 2 are obtained by applying the equations in the “theoretical bases” section to the early reflection portion of the experimentally-measured pressure and two-dimensional velocity room impulse responses. Thereafter, the sliding time window technique mentioned above is used to animate the changes in intensity with time.

Mm. 2. Animation showing a change in pressure and intensity over time in a rectangular room for the transient case with four frames showing various quantities of interest as described in Fig. 2.

Sound energy propagation is represented by pressure wavefronts emanating from the top left-hand corner where the source is located. As postulated, the wavefronts are broken up due to the interaction of reflecting sound waves from the adjoining walls. However, the propagation of sound energy from top to bottom is still evident, as is the deflection of intensity arrows by small, localized high-pressure zones (redder colors denote higher-pressure regions). Pockets of rotating intensity vectors moving across the room with time are indicative of vortical energy flow mixed with radial propagation.

Active and reactive intensity vectors do not change as rapidly as instantaneous intensity vectors by virtue of being time-averaged quantities. However, they still change gradually since the sliding time window method used to visualize them is sensitive to changes in intensity (time-averaged within window) from frame to frame. This is different from previous time-averaging measurement techniques where the measured intensity is averaged across a single measurement time frame only. Accordingly, the active intensity vectors follow the general direction of transient energy flow (represented by the instantaneous intensity vectors) around the room, albeit with some time lag. This again differs from the simulation of the steady-state axial vortex mode case in the preceding section where the instantaneous intensity moves in closed circular paths but the active intensity does not change.

Fig. 2. Energy flow in rectangular room for the transient case. From left to right: (1) instantaneous pressure and intensity, Eq. (3); (2) instantaneous intensity, Eq. (3); (3) active intensity, Eq. (8); and (4) reactive intensity, Eq. (9).
Continuing with the steady-state case, Mm. 3 is derived in the same way as Mm. 2, except this time, the experimentally-measured pressure and two-dimensional velocity room impulse responses are first convolved with white noise before the same sliding time window technique is used to capture the progression of intensity flow with time.

The same conclusions about the relationship between pressure and intensity can be drawn for the steady-state case as for the transient case. However, the pressure distribution seems to follow a more cyclical pattern than for the transient case. Interestingly, the instantaneous intensity and time-averaged active intensity animations are almost identical, an expected observation given that the energy flow within the room has stabilized for the steady-state case.

5. Conclusion

The method presented in this paper achieves the following main objectives: (1) It visualizes changes in intensity as a function of instantaneous pressure and velocity as sound energy propagates through a space for both the transient and steady-state cases in scale models. (2) It also obtains time-averaged quantities for pressure, velocity, and consequently, intensity from instantaneously-measured pressure and velocity data in the absence of solvable pressure and velocity distributions.

Future developments include the study of pressure and intensity distributions in more complex room geometries such as coupled spaces and free-form shaped rooms. While the use of scale modeling is not new in architectural acoustics, this method allows the acoustician to obtain a representative map of intensity flow progression in the equivalent actual-size room within a controlled environment. Considering that it took more than 24 h to complete the measurements for this scale model experiment, it is practically impossible to do likewise for an actual-size room in a real-world setting where extenuating factors such as temperature and noise are difficult to control for over extended lengths of time. This is due to the key difference between this method and existing scale modeling techniques in that it requires measurements on a fine grid across the entire cross-section of the room that is more comprehensive than measurements of a few sampled points in the same space—only then can the progression of intensity flow be accurately captured. This work could also be extended to studying the effect of different source positions on energy flow patterns within any space. Since transient intensity flows and tangential modal energy flows exhibit more complex patterns than that of steady-state axial modes, a denser measurement grid would more accurately capture the changes in flow directions, such as the temporal vortices seen in the animations. This method complements FEM modeling as a diagnostic tool in acoustical design and is especially relevant to small rooms for critical listening where the modal frequencies are within the normal range of human hearing. This in turn has potential applications in musical instrument acoustics, sound system design and many other elements that influence the quality of a recording.

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