Diffusion equation-based modelling of reverberation chambers for sound absorption measurements

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ABSTRACT
Since recent years, unacceptably large variations resulting from round-robin tests of the standardized measurements of random-incidence absorption coefficients have prompted research activities in architectural acoustics. This work investigates an estimation approach using a diffusion equation model (DEM) that is able to simulate both reverberant sound energies and energy flows inside reverberation chambers particularly for random incidence absorption coefficient measurements. The DEM also provides a high computational efficiency when compared with wave-based methods. The DEM analysis on the sound energy flows in a reverberation room further indicates violations of the diffuse field assumption around the boundary area of absorbing sample under test. Given the computational efficiency and higher order of the energy decay simulation of the DEM modeling, an approach to estimate absorption coefficient within the validity range of statistical room acoustics is investigated. The proposed approach estimates the absorption coefficients by evaluating both the DEM simulation results and the chamber-based measurement data. This paper examines sound energy flow fields of varied absorbing degrees of absorber samples in standardized sizes, discusses computational efficiency of the diffusion equation modeling for reverberant energy simulations in reverberation chambers, and considers potential significance applied to random-incident absorption coefficient measurements.

Keywords: Absorption coefficient, Diffusion Equation Model, FDTD, Reverberation Chamber

1 INTRODUCTION
One of recent research activities in architectural acoustics is now focused on the constancy of random incidence absorption coefficients obtained in reverberation chambers [1, 2]. However, controversy over the random incidence absorption’s correctness began decades ago. The concept of a diffused sound field, provided by Schroeder in 1959, is essential to the topic since it states that a room may be deemed entirely diffuse when the distribution of energy flow is uniform at a particular position. Any distribution pattern that isn’t totally uniform is therefore considered to be a “not completely diffuse” sound field. In this research work, consequently, the study concentration is to simulate such behavior of the sound energy distribution and discuss and analyze some of the representative distribution patterns.

2 THEORY
2.1 Diffusion Equation Model
The Brownian motion of particles in a fluid medium is described by the Diffusion Equation Model (DEM). The sound wave density, which is modeled in this study as straight-moving particles traveling at sound speed c, and the sound energy level, which causes a greater particle density, are analogous. The scenario might be viewed as air particles moving and collapsing in the air for the case of reverberation chamber measurement specified with volume V and surface area S, therefore we could develop the model using the following equations

\[
\frac{\partial w(r,t)}{\partial t} - D\nabla^2 w(r,t) = q(r,t) \text{ in } V,
\]
where \( w(r,t) \) represents the sound energy density in the volume, \( D \) stands for the diffusion coefficient determined by wavelength \( \lambda \) and sound speed \( c \)
\[
D = \frac{\lambda c}{3},
\]
and \( \nabla^2 \) is the Laplace operator.

### 2.2 Boundary Condition

Equation 1 is for media inside the volume only, which dictates the particle movement. As for the room surface, also called the boundary condition, the diffusion equation model uses this equation to explain how the particles will behave when they approach the boundary
\[
D \frac{\partial w(r,t)}{\partial n} + cAXw(r,t) = 0 \text{ on } S,
\]

(2)

The on-boundary condition factor \( AX \) in Equation 2 essentially describes how sound energy interacts with enclosed room borders; as a result, boundary condition factors will behave differently depending on the variance of the barriers’ absorption coefficients. We must make an assumption that the absorption coefficient is significantly larger than that in the Sabine equation since we are talking about the measuring method of acoustic absorption materials. This work uses the mixed-boundary condition factor with this presumption [3].

### 2.3 Finite Difference Scheme

Given the boundary conditions, finite difference approaches are the methods that are utilized to solve the DEM. The finite difference technique established in the three-dimensional sound field is used in this work [5, 6, 7]. The simplicity of such method is well recognized by not requiring or including any of Green’s functions, matrices, asymptotic functions, or basis functions. Furthermore, one simulation round could yield answers over a broad frequency range is also one of its great benefits.

The FD-DEM calculating procedure might be expressed as
\[
w_{i,j,k}^{n+1}(1 + \beta_0) = w_{i,j,k}^{n-1}(1 - \beta_0) - 2\Delta cmw_{i,j,k}^n
+ \beta_{0x} (w_{i+1,j,k}^n w_{i-1,j,k}^n)
+ \beta_{0y} (w_{i,j+1,k}^n w_{i,j-1,k}^n)
+ \beta_{0z} (w_{i,j,k+1}^n w_{i,j,k-1}^n),
\]

(3)

where the upper labels of \( w \) represents the time steps used by the model buffer before and after the current state, and their lower labels, correspondingly, state the spatial differences, with \( i, j, k \) taking care of each axis [5]. \( \Delta t \) is the resolution of each time step which could also be used for controlling total run time, \( m \) is the air absorption \( (m = 0.0022 \text{ for general cases, varies a tiny bit depending on humidity and temperature}) \), and
\[
\beta_{0x} = \beta_{0y} = \beta_{0z} = \left( \frac{2D\Delta t}{(\Delta v)^2} \right).
\]

(4)

When an object changes from one state of motion to another or from one state of matter to another, the boundary condition occurs in physics. However, the boundary condition in the context of acoustics, and specifically in this study, refers to the sound energy colliding with a wall or an absorbent medium, then reflecting or dissipating at contact or on the way back.

As for boundary conditions, the FD-DEM model used following equations to estimate them
\[
w_{0,j,k}^{n+1} = \frac{4w_{1,j,k}^{n+1} - w_{2,j,k}^{n+1}}{2\Delta x_D},
\]

(5)
and
\[ w_{n+1}^{L_x,j,k} = \frac{4w_{n-1}^{L_x,j,k} - w_{n+2}^{L_x,j,k}}{3 + \frac{2A_{n,j,k}}{D}}, \]  
(6)

where \( A_X \) represents the mixed-boundary condition mentioned above [3].

2.4 Absorption Estimation

The simulation batch contains two main functions: rendering energy flow animations, capable of both impulsive or steady-state, and deriving either an energy buildup function (ETF) or an energy decay function (EDC), depending on the method chosen. The purpose of creating energy flow animation is to showcase the non uniform distributed sound field, so called non diffuse. And after getting either the ETF or EDC from software simulation, we could use the measurement data and also calculate the Schroeder’s integration to get the energy decay curve. With the two decay curves, we could have them substracted and get the sum of squared error (SSE):
\[ SSE = \sum_{i=1}^{n} (y_i - f(x_i))^2, \]  
(7)

where \( y_i \) is the value to be predicted, or the real data; and \( f(x_i) \) the predicted data, or the simulation data.

The likelihood is evaluated from the SSE by using a Student-t distribution [4]
\[ L = \frac{\Gamma(K/2)}{2} \left( \pi \sum_{k=1}^{K} \epsilon_k^2 \right)^{-K/2}. \]  
(8)

Figure 1 is one flow chart aiding readers to understand the entire workflow.

3 RESULTS

Following figures showed simulation results in a rectangular chamber room with 498 cubic meters volume and the absorption panel shown in the figure has the (total) area of 10 square meters. In Figure 2, the energy flow
is captured at time step = 9 seconds, way longer than the designed empty chamber reverberation time. One can see that the energy flux arrows are pointing haphazardly in all directions, which means that the flows of energy do not have a definite and regular direction. Here according to the definition of diffuse sound field, it is safe to say that the energy flux is distributed uniformly throughout the entire chamber.

The likelihood function / posterior probability over time is plotted over possible absorption coefficient range at Figure 3.

4 DISCUSSION

When there is no absorption in the test chamber, the sound energy propagates completely randomly, or it can be understood that at any point, the sound energy is erratically dispersed in its own direction, we can confidently refer to the chamber sound field environment, particularly in Figure 1, as completely diffused. In fact, sound energy is dispersing extremely slowly, primarily due to air absorption, while still traveling everywhere.

However, when the sound absorber under test is present, the energy flow direction immediately demonstrates concentrated flow direction. Whether the absorption panel is uniform or divided, it tends to track the direction of the absorption source (Figure 4). When the absorption coefficient rises to an extraordinarily high number, like 0.99, the trend becomes pronounced even more. While experimentally Nolan et al. also experimentally demonstrated the violation of diffused sound field assumption and effectively displays such disparities, the DEM is also able to realistically reveal the substantial violation of diffuse sound field [8].

Using the energy flow animation from the DEM simulation, one can confidently draw the conclusion that once the sound absorber under test is placed in the testing chamber, the chamber will inevitably display its non-diffuseness as the energy flows are drawn to the absorbing panel, as was covered in the paper’s final section. This result may point to a potential issue with the current random incidence absorption coefficient measurement procedure: when attempting to apply the Sabine equation in the calculation steps, which assumed a fully diffused sound field, one may not be able to obtain correct results from a sound field that is not completely diffused.

Figure 5 exhibits the sound energy distribution of the chamber, and it is easy to conclude that in the scatter view, each individual point has its own strength shown in the color map, which means that compared with conventional predictions using the statistical room-acoustics simulation, the DEM has a higher statistical order, that allows the investigator to use any given point or any multiple points inside the chamber as the receiver.
position in a single simulation cycle. In that way, the computational load could be much lower especially for the chamber measurement simulation, considering that in current standards, multiple receiver positions are often used to ensure the accuracy. If the DEM is involved as the simulation part of the absorption examination, only one run is required for all of the actual receiver positions.

As for the meshing condition used in the simulation, considering the current standards’ minimum area requirement, a meshing condition based on mean free path (MFP) is selected but with the recommended oversampling of 16 times according to Trevor and Antonio [9] is actually used.

Because the likelihood always shows clear single peak in past examinations (Figure 3 for example), a searching algorithm could be applied to the estimation process, for further decreasing computational load. With the proper meshing condition set up for the regular rectangular shaped absorber sample in a testing chamber with standard volume, the average run time for each DEM simulation is a few seconds, and for all of the possible absorption value from 0 to 1, the searching algorithm will take around 20 times of DEM simulation to yield the final estimation, which means the entire estimation process will take only in the order of minutes, which is significantly faster than some room-acoustic simulation methods, and the author believes that with more advanced computing equipment and optimized coding, this process could be even faster.

5 SUMMARY

This paper discussed underlying problems of the current using random incident absorption coefficient testing standards and aimed to provide a solution with a DEM based simulation and Bayesian parameter estimation. The DEM simulates sound energy distribution inside the reverberation chamber and is able to provide results accurately and efficiently compared with other room-acoustic simulation methods. The energy flow discussed in the paper shows obvious sign of non-diffuseness, and also proved the ability of the DEM for simulating such of an environment. The Bayes parameter estimation is then made possible, and could become a solution with great potential to yield an accurate absorption coefficient measurement within minutes of computation. Further discussion based on detailed meshing condition selection and Bayes parameter estimation process is expected, the searching algorithm is also awaiting to be optimized for saving computation time.
Figure 4. The energy flows in an standard testing chamber, the wall absorption is set to be 0.02, and the material being tested (blue boxes) has the absorption coefficient of 0.99. Red dot at upper right side represents the sound source.

Figure 5. A sound energy distribution plot of one example DEM simulation, with a slice view from the X-Z axis.
REFERENCES