The standing wave ratio method discussed in Sec. 2.3 represents a classical technique to determine acoustic reflection properties of a material under test in the tube. One of its drawbacks is that the measurement can only be conducted under single-tone excitations. There exist a number of advanced measurement techniques using so-called impedance tubes or Kunt’s tubes for the normal incidence measurement of surface reflection coefficient, characteristic impedance of porous materials, and sound transmission loss of a material under test. While advanced methods essentially employ a transfer function measurement method for broadband determination of acoustic reflectance/impedances, there are exist many limitations. Major constrains of the tube-based method can be overcome by freefield or quasi-freefield measurement techniques, they are even suitable for in-situ measurements. This Chapter introduces some of these advanced methods.

6.1 Tube-based Transfer-Function Methods

Long and narrow tubes facilitate plane wave propagations. Microphones deployed at two, three or more positions near a material under test can be used for acoustic transfer function measurements to determine normal incident surface properties including acoustic reflectance, characteristic impedance of the material under test. This Section introduces two and three microphone methods.

6.1.1 Two-Microphone Transfer-Function Method

Figure 6.1 schematically illustrates a segment near one end of a long and thin tube where a material under test is properly mounted at the tube termination. Two microphones are placed at positions labeled by ’1’ and ’2’, separated by
distance \( s \), also termed *separation* in short.\(^1\) They sense sound pressure signals, expressed in the frequency domain as \( P_1 \) and \( P_2 \). When acoustic wavelength is much larger than the diameter of the tube \( \lambda \gg D \), the acoustic wavefront is the same across each individual cross-section labeled by ‘1’, ‘2’ and ‘m’, respectively. The distance between position ‘1’ and ‘m’, namely between the microphone ‘1’ and the surface of the material is known to be ‘\( L \)’.

![Diagram showing normal incidence plane wave for acoustic reflection coefficient measurements using two microphones](image)

**Fig. 6.1.** Normal incidence plane wave for acoustic reflection coefficient measurements using two microphones [8, 9]. A transfer function \( H_{12} \) is defined between microphone position 1 and 2, separated by distance \( s \).

### Surface reflectance (reflection coefficient)

Both incident and reflected pressure components are conceivably existing at position ‘1’, ‘2’, and ‘m’. The medium is considered lossless. If an incident wave component travels from position ‘1’ to position ‘2’ (forward-propagating) as illustrated in Figure 6.1, the sound pressure in frequency domain at position ‘2’ can be expressed as

\[
P_{2,i} = P_{1,i} e^{-j\beta s},
\]

where \( P_{1,i} \) and \( P_{2,i} \) represent the incident components of sound pressures in the frequency domain, and \( s \) represents separation distance of the two microphones. When the reflected wave component travels from position ‘2’ to position ‘1’, it can be written as

\[
P_{2,r} = P_{1,r} e^{j\beta s}.
\]

where \( P_{1,r} \) and \( P_{2,r} \) represent the reflected components of sound pressures at the two positions in the frequency domain. In similar fashion, the sound pressure components between position ‘1’ and position ‘m’ as illustrated by Figure 6.1 are expressed by

---

\(^1\) The method was first introduced by Chung and Blaser [8, 9] using a time-domain explanation. The method introduced in this Chapter explains the theory in frequency domain.
\[ \begin{align*}
\mathbf{P}_{m,i} &= \mathbf{P}_{1,i} e^{-j\beta L}, \\
\mathbf{P}_{m,r} &= \mathbf{P}_{1,r} e^{j\beta L}.
\end{align*} \tag{6.3} \]

Between two microphones at position '1' and '2', the two sound pressure signals \( \mathbf{P}_{1}, \mathbf{P}_{2} \) in the frequency domain build a transfer function, introduced in Sec. 1.6 as

\[ \begin{align*}
\mathbf{H}_{12} &= \frac{\mathbf{P}_{2,i} + \mathbf{P}_{2,r}}{\mathbf{P}_{1,i} + \mathbf{P}_{1,r}} = \frac{\mathbf{P}_{1,i} e^{-j\beta s} + \mathbf{P}_{1,r} e^{j\beta s}}{\mathbf{P}_{1,i} + \mathbf{P}_{1,r}}, \tag{6.5}
\end{align*} \]

where Equations (6.1) and (6.2) come to use here in the nominator on the right-hand side. Rearranging

\[ \begin{align*}
\mathbf{H}_{12} (\mathbf{P}_{1,i} + \mathbf{P}_{1,r}) &= \mathbf{P}_{1,i} e^{-j\beta s} + \mathbf{P}_{1,r} e^{j\beta s}, \tag{6.6}
\end{align*} \]

\[ \begin{align*}
\left( \mathbf{H}_{12} - e^{j\beta s} \right) \mathbf{P}_{1,r} &= \left( e^{-j\beta s} - \mathbf{H}_{12} \right) \mathbf{P}_{1,i}. \tag{6.7}
\end{align*} \]

Using Equations (6.3), (6.4), and (6.7), the surface reflectance at the front surface of the material 'm' as in Fig. 6.1 is defined as the reflected component vs. the incident component, which can be expressed by

\[ \begin{align*}
\mathbf{R}(f) &= \frac{\mathbf{P}_{m,r}}{\mathbf{P}_{m,i}} = \frac{\mathbf{P}_{1,r}}{\mathbf{P}_{1,i}} e^{j2\beta L} = \frac{\mathbf{H}_{12}(f) - e^{-j\beta s}}{e^{j\beta s} - \mathbf{H}_{12}(f)} e^{j2\beta L}, \tag{6.8}
\end{align*} \]

where the reflectance (reflection coefficient) \( \mathbf{R}(f) \) and the transfer function \( \mathbf{H}_{12}(f) \) from Equation (6.5) are explicitly denoted as functions of frequency, particularly,

\[ \mathbf{H}_{12}(f) = \frac{\mathbf{P}_{2}(f)}{\mathbf{P}_{1}(f)}. \tag{6.9} \]

The normal incidence surface impedance can be derived from the reflection coefficient in Equation (6.8),

\[ \mathbf{Z}_{s} = \rho c \frac{1 + \mathbf{R}}{1 - \mathbf{R}}, \tag{6.10} \]

with \( \rho c \) being the characteristic resistance of air, \( \rho \) is air density, and \( c \) is sound speed. The normal incidence absorption coefficient is determined by

\[ \alpha = 1 - |\mathbf{R}|^{2}. \tag{6.11} \]
Valid frequency range

Equation (6.8) expresses the reflection coefficient as a function of frequency, determined through the transfer function $H_{12}(f)$. Notably, the fraction in Equation (6.8) will become

$$\frac{H_{12} - e^{-j\beta s}}{e^{j\beta s} - H_{12}} = -1$$

at $e^{-j\beta s} = e^{j\beta s}$, when

$$\beta s = n\pi \quad \text{with} \quad n = 1, 2, 3, \ldots ,$$

which represents singular conditions for this method [8].

With $n = 1$ and $\beta = 2\pi f/c$ the microphone separation $s$ can be determined upon a limit frequency for the tube measurement to avoid the singular conditions

$$s < \frac{c}{2f} = \frac{\lambda}{2},$$

where $f$ represents the limit of the valid frequency, and $\lambda$ is the wavelength at frequency $f$. This relation is consistent with the spatial Nyquist frequency applied in the principle of finite difference to approximate gradient (see Sec. 6.2.4).

In addition, the normal incidence measurement requires a plane wave condition, which is

$$f_h < \frac{K \cdot c}{d},$$

with $d$ being the diameter of the circular cross-section tube, where $K = 1$. For square tubes with $d$ being their diagonal length, $K = 0.5$. Equation (6.15) is decisive to determine the higher limit frequency $f_h$, assuming that the tube is long enough. Given the higher limit frequency dictated by the tube diameter, a lower bound of the microphone separation $s$ can be determined. In order to ensure the separation being sufficiently smaller than half wavelength given in Equation (6.14), the recommended rule of thumb [13] is to take $4/5$ of that in the singular condition,

$$s_{\text{small}} \leq \frac{4\lambda_h}{5} = \frac{2\lambda_h}{5},$$

while the tube should be valid for the frequency range towards lower end at which, the separation has to be selected [13] at least $1\%$ of the longest wavelength in order to be able to rely on the measured differences between the two microphone positions.

$$s_{\text{large}} > 0.01\lambda_l.$$

(6.17)
The lower limit frequency is often dictated by the length of the tube. It needs sufficient long in order to ensure sound waves becoming plane wave when reaching the microphones and the material under test.

Absorbing wedges are often inserted at the opposite end near the sound source. Considering the frequency range near the lower frequency limit, even a careful selection of absorbing materials may not meet the need of total absorption reaching as high as \( \alpha > 0.98 \), and the separation has to follow the condition in Equation (6.17), there will still be some reflection residuals negatively impacting the measurement accuracy. For this reason, the lower frequency limit \( f_1 = c/\lambda_1 \) should be dictated by capability of eliminating these reflection residuals. Equations (6.16) and (6.17) indicate that there exists a range of the separation distance

\[
s_{\text{small}} \leq s \leq s_{\text{large}}
\]

in order to cover a sufficient bandwidth which fulfills at the same time the singular condition given by Equation (6.14). To this end a combination of different separations is often exercised.

**Fig. 6.2.** Possible implementation of the transfer function method using one microphone. Sequential measurements at two microphone positions synthetically implement the two-microphone transfer function method.

Two-microphone transfer function method discussed so far, requires two simultaneous microphone channels. This requires highly accurate phase-match of the two microphones, even the magnitude responses of microphone sensitivities are flat enough. Small mismatches in phases or in sensitivity responses will lead to large measurement errors and uncertainties. With the advent correlation technique (as introduced in Chapter 4), the transfer functions and the impulse responses of acoustic devices / systems can be experimentally measured with high accuracy of phase-locking (synchronization). This enables sequential measurements using single microphone in order to avoid highly accurate phase-matching of two or multiple simultaneous microphone channels [7]. Figure 6.2 illustrates one possible implementation using a single microphone in terms of sequential measurement at two or more microphone positions, where the advanced measurement techniques discussed in Chapter 4 have to be used. In particular, the impulse responses between the sound source at one end of
the measuring tube and the microphone at position '1' and '2' can be sequentially measured. The transfer function in Equation (6.9) is then determined using fast Fourier transforms. In order to cover both low frequency and high frequency ranges, some additional microphone positions to ensure small or large separations need to be measured. Recent effort has also been made to extend the upper limit frequencies through additional measurements at many microphone positions [3].

6.1.2 Three-Microphone Transfer-Function Method

The two-microphone method discussed previously yields measurement results of normal incidence surface reflection coefficient, absorption coefficient or the normal incidence surface impedance, that cannot directly characterize the materials. This Section introduces one of advanced methods suitable for characterization of fibrous and porous materials [22], commonly used for a wide variety of absorbing materials in acoustics applications and noise control engineering. The technique is essentially an extension of the two-microphone method under plane wave assumptions. Figure 6.3 schematically illustrates the tube configuration for determining the characteristic impedance $Z_c$ and the complex-valued propagation coefficient $\gamma$ of porous materials or those materials which can be considered as equivalent fluid media.

One more microphone is placed at position '3' in the rigid backing right on the back surface of the porous material as shown in Figure 6.3. Two microphones at position '1' and '2', separated by a distance $s$ are placed in front of the material surface with microphone '1' being positioned at $L$ distance from the material surface, similar to that in Figure 6.1. The porous material under test has a known thickness $d$. The measurement technique experimentally determines its characteristic impedance $Z_c$ and propagation coefficient $\gamma$. As before, Equation (6.8) yields the reflection coefficient $R$ at the front-side, when the microphone separation $s$ and distance $L$ to the material front surface are known.

Fig. 6.3. Three microphone transfer function method for characterizing porous materials [22]. The medium inside the tube is considered as lossless, with its characteristic resistance $\rho c$ and its propagation coefficient $\beta$. 
Denoting the sound pressure $P_3$ and particle velocity $V_3$ at position '3', the sound pressure is measured by microphone at position '3'. The sound pressure $P_m$ and particle velocity $V_m$ at the material front surface can then be expressed [27] as:

$$\begin{bmatrix} P_m \\ V_m \end{bmatrix} = \begin{bmatrix} \cos \gamma d & j Z_c \sin \gamma d \\ j \sin \gamma d/Z_c & \cos \gamma d \end{bmatrix} \begin{bmatrix} P_3 \\ V_3 \end{bmatrix}. \quad (6.19)$$

At the rigid termination $V_3 = 0$ so the characteristic impedance $Z_c$ can be derived from the above equation in terms of the surface impedance $Z_s = P_m / V_m$ and the propagation coefficient $\gamma$ when the material thickness $d$ is given,

$$Z_c = j Z_s \tan \gamma d, \quad (6.20)$$

where the surface impedance $Z_s$ is determined by Equation (6.10), via Equation (6.8) through experimentally measured pressure spectra from the two microphones at position '1' and '2', in front-side of the tube as discussed in Sec. 6.1.2. From Equation (6.19), it also follows that

$$P_m = P_3 \cos \gamma d, \quad (6.21)$$

resulting in the complex-valued propagation coefficient

$$\gamma = \cos^{-1}(H_{3m}) / d, \quad (6.22)$$

with

$$H_{3m} = \frac{P_m}{P_3} = \frac{P_m P_1}{P_1 P_3} = \frac{P_m}{P_1} H_{31}, \quad (6.23)$$

and

$$H_{31}(f) = \frac{P_1(f)}{P_3(f)}. \quad (6.24)$$

Similar to $P_1$ as discussed previously, $P_3$ represents the sound pressure in frequency domain measured by the microphone at position '3'.

The sound pressures $P_1$ and $P_m$ represent superpositions of the incident and reflected components

$$P_1 = P_{1,i} + P_{1,r} = P_{m,i} \left( e^{j \beta L} + Re^{-j \beta L} \right), \quad (6.25)$$

$$P_m = P_{m,i} + P_{m,r} = P_{m,i}(1 + R), \quad (6.26)$$

Complex division of Equation (6.26) by Equation (6.25) leads to

$$\frac{P_m}{P_1} = \frac{1 + R}{e^{j \beta L} + Re^{-j \beta L}}. \quad (6.27)$$
A substitution of Equation (6.27) into Equation (6.23), and then into Equation (6.22) leads to the complex-valued wavenumber containing the propagation coefficient

$$\gamma_d = \cos^{-1}\left( \frac{1 + R}{e^{j\beta L} + R e^{-j\beta L} H_{31}} \right),$$

(6.28)

where the transfer function $H_{31}$ can be measured via microphones at position '1' and '3', as in Equation (6.24) through the pressure spectra, while $R$ is determined using Equation (6.8). Note that the complex-valued wavenumber $\gamma_d$ containing the complex-valued propagation coefficient $\gamma$ of the material and its thickness $d$ are used in Equations (6.19)-(6.21). A substitution of Equation (6.28) into Equation (6.20) yields the characteristic impedance $Z_c$.

Using three microphones, two on the up-stream (source) side near the sample material, and one right at the back surface of the material under test, the characteristic impedance and the propagation coefficient can be experimentally determined which relies also on intermediate step of getting the acoustic surface impedance of the material under test [in Equation (6.10) via Equation (6.20) and Equation (6.28)]. The valid frequency range and the singular conditions as presented in Sec. 6.1.1 hold in general for the three-microphone method. To cover a wide frequency range, a combination of different microphone separations is a common practice as well, as discussed using Equations (6.16)-(6.18). The three-microphone method is also advantageous over the four-microphone method [23] (two microphones on the up-stream side and two on down-stream side). Not only one channel/measurement position less needs to be involved in the measurement procedure, but the microphone right on the back surface will measure enhanced signals due to the rigid termination, in comparison with those on the down-stream side of the four-microphone method.

Similar to the two-microphone method, the phase-accurate correlation technique can be applied in sequential measurements using one single microphone at three microphone positions. As mentioned before, the sequential measurements at different microphone positions are intended to avoid the measurement errors of microphone mismatches [7].

### 6.1.3 Sound Speed and Dissipation Estimations

A critical issue is related to microphone positions [4], particularly, microphone separation $s$, and microphone distance $L$ in Equations (6.8) and (6.28), as also shown in Figures 6.1 and 6.3. They sensitively impact the tube measurement accuracy. Instead of accurate, direct measurements using rulers or calipers, which are also contaminated with uncertainties, since equivalent acoustic centers of the microphones are hardly to be exactly measurable [15]. More profoundly, the sound speed is constantly drifting with slow changing of ambient environment. A recent effort has applied a model-based Bayesian method [7] to infer microphone positions, the sound speed and the dissipation due to
so-called boundary effect of tube inner walls through one calibration measurement at a rigid termination without the material under test as shown in Fig. 6.4.

The surface impedance of the air layer of thickness \( d \) in front of a rigid backing is determined, similar to Equation (6.20), by

\[
Z_s = -j \varrho c \cot(\beta d). \tag{6.29}
\]

Solving for the reflectance using Equation (6.10) leads to a prediction model of the air layer reflectance \( R_M \) of thickness \( d \) [7],

\[
\frac{j \cot(\beta d) + 1}{j \cot(\beta d) - 1} = R_M = \frac{j \cot(\omega t_d) + 1}{j \cot(\omega t_d) - 1}, \tag{6.30}
\]
where \( t_d = d/c \), so as for the model on the right-hand side to include the sound speed as a pending parameter. Figure 6.5 illustrates the air layer reflectance predicted by this model in its real- and imaginary parts as function of frequency for four different layer thicknesses.

Using Equation (6.8), the two-microphone transfer function measurement in the tube setup as shown in Fig. 6.4 yields experimental reflectance data when the microphone positions are optimally estimated via an inverse process [7] as illustrated in Fig. 6.6 (a), (b).

![Fig. 6.6. Comparison of complex-valued reflectances between measured data and model-predictions. [7] (a), (b) For an air layer of 1.5 cm and 2 cm without involving the dissipation of the tube walls. (c), (d) For an air layer of 1.5 cm and 2 cm involving the dissipation of the tube walls.](image)

Note that the microphone positions fixed in tube devices are necessarily calibrated only once, but the sound speed of medium air is changing constantly, due to slow drift of the ambient environment. Chen et al. [7] parameterize Equation (6.8) such that the sound speed along with the microphone positions becomes pending parameters. These are simultaneously estimated within the same model-based inversion framework.

Equation (6.8) is valid for calculating the reflectance in lossless media. However, due to the dissipation in the medium air and the boundary effect of the tube walls (see Chapter 11 in [27]), there is always some energy loss during the sound propagation in narrow tubes. For weak losses, the phase coefficient \( \beta \) can be replaced with a complex-valued propagation coefficient \( \gamma \),
6.1 Tube-based Transfer-Function Methods

Fig. 6.7. Damping coefficients for a tube made of PVC pipe of 5 cm in diameter under different temperatures at 18.7°C, 20.3°C, and 22.2°C. The scale factor \( \zeta = 4.83, 3.61, 3.17 \) are estimated [7].

\[
\gamma = \alpha_\zeta + j \beta = \alpha_\zeta \frac{c + j \omega}{c},
\]

(6.31)

where \( \alpha_\zeta \) represents a frequency-dependent damping coefficient. For calculating the reflectance, Equation (6.8) is expressed for lossy case as

\[
R_D = \frac{H_{12} - e^{-\gamma_s}}{e^{2\gamma_s} - H_{12}} - \frac{H_{12} - e^{-(\alpha_\zeta c + j \omega) t_s}}{e^{(\alpha_\zeta c + j \omega) t_s} - H_{12}} e^{2(\alpha_\zeta c + j \omega) t_L},
\]

(6.32)

with \( t_s = s/c \) and \( t_L = L/c \). In the tube measurements under certain frequency limit (on order of kHz) the damping loss is primarily attributable to the dissipation caused by the boundary effect of tube walls, it can be predicted through a so-called ‘wide tube’ dissipation model [see Eq.(11.9) in Chapter 11 of [27]],

\[
\alpha_\zeta = 6.7 \times 10^{-6} \frac{U}{A} \sqrt{\omega} = 9.48 \times 10^{-6} \frac{\zeta \sqrt{\omega}}{r},
\]

(6.33)

where \( U, A \) represent the circumference and the cross-sectional area of the tube, respectively. Quantity \( r \) is the radius of the circular tube. The dissipation model is additionally parameterized with a scale parameter \( \zeta \) to include possible variations due to different tube wall materials, it also copes with ambient environment changes.

The experimental data in Equation (6.32) derived from the transfer function method in the tube measurement are now parameterized by two additional parameters \( c, \zeta \) when the microphone positions are well estimated and calibrated. Figure 6.7 illustrates the dissipation as a function of frequency caused by the boundary effect of the tube walls, the scale parameter \( \zeta = 4.17 \)
is estimated under the ambient condition of $18^\circ \text{C}$. When determining the reflectance $R_D$ using Equation (6.32) to include the dissipation, the measurement results match better the prediction model $R_M$ as shown in Figure 6.6 (c), (d).

The calibration measurement should be undertaken immediately before or after actual measurements of materials under test, so that the critical parameters ($s, L, c, \zeta$) inversely estimated from the model-based estimation are assigned to Equation (6.32) for determining the material reflectance.

### 6.2 Free-Field Methods

The acoustic surface reflectance is also defined as

$$\begin{align*}
R(f) &= \frac{P_r(f)}{P_i(f)} = \frac{\text{FT}[p_r(t)]}{\text{FT}[p_i(t)]},
\end{align*}$$

where $p_i(t)$ and $p_r(t)$ represent incident and reflected pressure signals in time domain at the front surface of the material under consideration, operator $\text{FT}[\cdot]$ stands for Fourier transform, while $P_i(f)$ and $P_r(f)$ are pressure spectra for incident and reflected pressure signals, respectively. Note that the definition of reflectance in Equation (6.34) implicitly assumes a plane wave condition, restricting to the plane wave reflectance (spherical wave reflectance will be clearly different). Equation (6.34) also hints at an experimental approach in free-field if the pressure signals, either in time-domain or spectral signals in frequency domain can be obtained. Ideally the measurement needs to be carried out in free-field, such as in an anechoic chamber in laboratories. In practice, it is also desirable to carry out the measurement in quasi-free-field, such as in large spaces with the materials already built into the site or already in use. This kind of measurements so-called \textit{in-situ}, requires careful arrangement near the material sample under test, this Section discusses two approaches.

#### 6.2.1 \textit{In-situ} Windowing Method

Figure 6.8 schematically illustrates one possible configuration [12]. In Figure 6.8 (a) a microphone is placed between a sound source (loudspeaker) and a large material panel under test with a perpendicular distance $d$ to the material surface. The sound source is placed with a perpendicular distance $D$ to the material surface. When measuring an impulse response of this measurement setup, Figure 6.8 (b) schematically illustrates a straightforward windowing approach to separating the incident impulse response segment $h_i(t)$ and reflected impulse response segment $h_r(t)$ contained in an \textit{in-situ} measurement of the impulse response of this setup [12]
6.2 Free-Field Methods

Fig. 6.8. Free-field measurement method for normal incidence reflectance measurement [12]. (a) Measurement setup with a sound source and a microphone relative to material panel under test. (b) Time trace of idealized echogram, indicating the timing relationship with dash-line windows used to separate the incident and reflected pulses.

\[ h_m(t) = \underbrace{h_i(t - t_0)}_{\text{direct sound}} + \underbrace{h_r(t - t_0 - \tau_r)}_{\text{surface reflection}} + n(t) + \underbrace{h_s(t - t_0 - \tau_s)}_{\text{speaker reflection}} + \sum_u K_u h_r(t - t_0 - \tau_u), \tag{6.35} \]

where

\[ t_0 = \frac{D - d}{c}, \quad \tau_r = \frac{2d}{c}, \quad \text{and} \quad \tau_s = \frac{2D}{c}. \tag{6.36} \]

\( K_u \) represents coefficients of unwanted reflections from irrelevant surfaces, or objects in the measurement site. Time function \( n(t) \) is background noise. In anechoic environment, the last sum-term on the right-hand side of Equation (6.35) would technically disappear. However, during the measurement in-situ, it is possible to make \( \tau_u \geq \tau_s \) through adjustment of the speaker distance to the material sample under test and the material panel size, so that
a straightforward application of two window functions as sketched in Figure 6.8 (b) will be able to extract two impulse response portions at the given microphone positions. Considering the acoustic surface reflection as a linear time-invariant system with a plane wave reflection impulse response \( r_R(t) \), it follows \([12]\)

\[
\frac{1}{D - d} h_i(t - t_0 - \tau_r) \approx \frac{r_R(t)}{D + d} * h_i(t - t_0),
\]

(6.37)

with symbol, '*' denoting linear convolution, and \( \tau_r \) represents the delay time caused by a round-trip distance from microphone to the material surface and back, and ‘\( \approx \)’ implies that the background noise \( n(t) \) is ignored here and throughout the following discussion.

Fourier transforming the both sides of Equation (6.37) yields

\[
\frac{D + d}{D - d} H_r(f) e^{-j2\pi f(t_0 + \tau_r)} = R(f) H_i(f) e^{-j2\pi f t_0},
\]

(6.38)

leading to the reflectance as a function of frequency

\[
R(f) = \frac{D + d}{D - d} \frac{H_r(f)}{H_i(f)} e^{-j2\beta d},
\]

(6.39)

with \( R(f) = \text{FT}[r_R(t)] \), \( \beta = 2 \pi f/c \), \( \tau_r \) in Equation (6.36) comes into use. The ratio \( (D + d)/(D - d) \) represents a path-length correction factor.

### 6.2.2 In-situ Subtraction Method

Figure 6.9 (a) schematically illustrates an alternative method for the surface reflectance measurement. One microphone is placed near the surface of test sample panel, along the perpendicular axis from the sound source to the sample surface. The major difference to the windowing method previously discussed in Sec. 6.2.1 is that the reflected signal from the surface is moved towards the direct sound signal as schematically shown in Figure 6.9 (c), while the direct sound signal and the reflected signal from the loudspeaker-face are remained the same if the loudspeaker distance to the sample surface remains the same.

The method is more suitable for in-situ applications \([16, 18]\), it inherently extends the time window width, therefore, extending the valid frequency range towards the lower limit as will be discussed in the following in Sec. 6.2.3. The method is more challenging than the direct windowing method, the in-situ measurement yields an impulse response, which can also be expressed in Equation (6.35), but with

\[
t_0 = \frac{D}{c}, \quad \tau_r = \frac{2 d}{c}, \quad \text{and} \quad \tau_s = \frac{2 D}{c}.
\]

(6.40)

The profound difference lies in an extremely short delay time \( \tau_r \) due to a small distance \( d \) to the material surface, often on order of a few millimeters
as indicated in a magnified sketch in Figure 6.9 (b), as the reflected signal falls within the time range of the direct sound impulse response. Unlike the windowing method as discussed in Sec. 6.2.1, a straightforward separation of the two signals \( h_i(t-t_0) \) and \( h_r(t-t_0-\tau_r) \) in terms of windowing is not possible any more. Figure 6.9 (c) conceptually illustrates a windowing function with window width of approx. \( 2D/c \). Time segment over \( 2D/c \) would only contain a composition of the direct sound and the surface reflection impulses

\[
h_m^{(w)}(t) = h_i(t-t_0) + h_r(t-t_0-\tau_r),
\]

where the two components on the right-hand side are temporally overlapped with each other. Figure 6.10 (a) illustrates one example of experimentally measured reference and material impulse responses.

Mommertz [16] suggested to conduct a reference measurement with the sample material being absence, in addition to the material measurement with the microphone near the material surface. The intension is to obtain identical direct sound impulse response through the reference measurement,

\[
h_{\text{ref}}^{(w)}(t) = h_i(t-t_0),
\]

where the reference measurement could also be conducted in-situ, and the same windowing as used in Equation (6.41) can be applied to the measured
Fig. 6.10. Impulse response segments from in-situ experimental measurements. (a) Impulse response segment measured in front of the material panel along with that from reference. (b) Impulse response after the direct-sound removal along with the reference impulse response. Both signals are also windowed using a right-half Hanning window as shown in Figure 6.12. (c) Magnified view of the segments in the dot-line framed area within (b), showcasing the reflected impulse has been successfully resolved from the measured segment as shown in (a).
impulse response to isolate the windowed direct sound impulse as the reference $h^{(w)}_{\text{ref}}(t)$.

The critical issue is that the relative position/distance between the loudspeaker and the microphone has to be strictly unchanged during the both measurements, so that the measured signal in front of material can be processed so as to subtract the direct sound signal using Equation (6.42) from the composition of the direct sound and the reflected signal in Equation (6.41) contained in the material measurement result. A subtraction of Equation (6.41) by Equation (6.42) yields

$$h^{(w)}_m(t) - h^{(w)}_{\text{ref}}(t) = h_r(t - t_0 - \tau_r). \quad (6.43)$$

Particularly for in-situ measurements, a dedicated fixture, stably holding the microphone relative to the loudspeaker, is highly required. The reference measurement needs to be conducted without the material being in front of the microphone. This can be carried out when the loudspeaker-microphone setup is facing to empty space, or facing to the space where the unwanted obstacles are far away. Advanced correlation techniques of impulse response methods [7] need to be applied so as to conduct sequential measurements of the reference and material impulse responses. The sequential measurements should be conducted within as a short time interval as possible so that any possible ambient environment changes will insignificantly impact accuracy of the attempted identical signals, since the time-variance between the two measurements will negatively impact the subtraction results [16, 21].

Despite of all careful conduction of both measurements in sequential manner, experimental practice has indicated that a straightforward subtraction in Equation (6.43) would rarely yield satisfactory results [16, 21]. Ambient environment changes, such as, temperature, and medium drifts inevitably make the two sequential measurements of the reference signal $h^{(w)}_{\text{ref}}(t)$ in Equation (6.42), and direct sound component $h_i(t - t_0)$ in Equation (6.41) contained in material signals, not accurately identical. To implement a subtraction successfully, an numerical over-sampling scheme is recommended, unless the measurements were conducted already at a sufficiently over-sampled frequency, which requires highly demanding measurement resources (such as at a sampling frequency between 315 kHz and 1 MHz).

Figure 6.11 recommends a numerical over-sampling scheme, summarized as follows

- **Up-sampling**: the two impulse responses from both the material and the reference measurements (often at an audio sampling frequency) are up-sampled by a factor ranging from 8 to 32 [21]
- **Alignment**: In the up-sampled domain, a temporal and an amplitude alignment of the material impulse response with respect to the up-sampled reference impulse response is undertaken if the refined differences of direct sound portion can be recognized
Fig. 6.11. Flow diagram of over-sampled subtraction for obtaining the acoustic reflectance, $\hat{R}(f)$.

- **Subtraction**: in the up-sampled domain, the subtraction similar to Equation (6.43) is carried out.
- **Down-sampling**: the subtracted result is still in the up-sampled domain, a down-sampling by the same factor is applied to the subtracted result, leading to the reflection impulse response from the material surface in the original sampling domain.
- **Windowing**: the same window function (a right-half window is recommendable) is applied to both the material reflection impulse and followed by possible zero-padding.
- **FFT / Division**: the zero-padding makes the two signals to be featured by a FFT-length (in form of $2^m$ points), so as to perform the fast Fourier transforms of the two signals followed by complex-valued division of the reflection spectrum by the incidence spectrum similar to Equation (6.34).

Figure 6.10 (b) and (c) illustrate a so-subtracted impulse response from the experimentally measured reference and the material impulse responses. In particular, Figure 6.10 (c) provides a magnified view of comparison between the subtracted impulse (due to the material reflection) and the reference (direct sound) impulse. The subtraction process (over-sampling/subtraction/down-sampling) largely removes the direct sound component $h_i(t - t_0)$, only an insignificant residual left to be seen in this magnified view. Robinson and Xiang [21] have applied an optimization strategy to further improve the subtraction accuracy.
6.2.3 Windowing, Limit Frequency, Source and Receiver Positions

Both the windowing method in Sec. 6.2.1 and the subtraction method in Sec. 6.2.2 require windowing. The window width $T_W$ in time is decisive for the lower limit frequency of data analysis, in general, and of the reflectance measurement, in particular as discussed in this Chapter. The lower limit frequency $f_L$ is reciprocal to the window width [12]

$$f_L = f_\Delta = \frac{1}{T_W}. \quad (6.44)$$

Fig. 6.12. Relationship between the window width $T_W$, frequency resolution $f_\Delta$, and the lower limit frequency $f_L$ of the windowing method and the subtraction method.

To shed light to this reciprocal relation, Figure 6.12 illustrates the data points in discrete frequency domain, assuming the impulse response segment, either $h_i$, and $h_r$ as illustrated within the dash-line window frames in Figure 6.8 or $h_{mW}(t)$ within the dash-line window frame in Figure 6.9 (c). These windowed impulse response segments carry useful information limited by the window width, expressed in time by $T_W$. The discrete frequency axis in Figure 6.12 highlights the discrete Fourier transform of the windowed impulse response segments, in form of data dots. Each is separated by the frequency interval / resolution $f_\Delta = 1/T_W$. The first two data dots represent the both data points at 0-frequency and at the $f_\Delta$-frequency, below the first $f_\Delta$-frequency the data contain no useful spectral content of the respective impulse response. Since the second data point at frequency of the first $f_\Delta$ begins to carry the spectral content of the measured data. According to Equation (6.44), the lower limit frequency of the in-situ windowing method as shown in Figure 6.8 (b) is

$$f_L \geq \frac{c}{2d}, \quad \text{given} \quad D \geq 2d, \quad (6.45)$$

and the lower limit frequency of the in-situ subtraction method in Figure 6.9 (c) is

$$f_L \geq \frac{c}{2D}. \quad (6.46)$$
Fig. 6.13. Window functions used to separate the useful impulse response portions. (a) Symmetrical Black-Harris and Hanning windows. (b) Right-half Black-Harris and Hanning windows with a plateau before the start of the half window functions.

Figures 6.8 and 6.9 illustrate that the impulse responses are assumed to be direct measurement results for decisive processing steps, they need to be extracted from the entire impulse responses by windowing, particularly in-situ, where the entire room impulse responses are primarily the raw data. There exist a number of window functions, often in symmetrical forms as exemplified in Figure 6.13 (a). For these impulse responses which are often featured with steep on-sets in responding to the direct sound arrival, the classical windowing functions in symmetrical forms are less suitable, rather those in right-half window forms as in Figure 6.13 (b). The top value of the window function is equal to 1.0, it is reasonable to extend the right-half window to feature a 'flat-
top’ (plateau). For the methods discussed in this Chapter, window functions with zero-value at the end, such as Hanning window or Black-Harris window are recommended, as they favor zero-padding after windowing.

Note that zero-padding after the right-half windowing, may prolong the data length, however, the zero-padding definitely doesn’t enrich the frequency content of the windowed impulse response at all, therefore, has no effect on the lower limit frequency $f_L$. The only change is to fill up additional interpolating points between the original data points as illustrated in Figure 6.12. The changes also include spectral changes on the material reflection impulse and the reference impulse, since the multiplicative manipulations (windowing) of the impulse response segments in time-domain correspond to convolution of the window spectrum with the spectra of the measured impulse response segments in frequency-domain. But these changes are less violative than the rectangular windows, particularly the complex division as indicated in Equation (6.34) largely removes these spectral changes caused by windowing. For this reason, the windowing functions applied to the two different impulse response segments need to be the same length and at relatively the same temporal locations.

The speaker distance should not be too far away from the material under test, it will require the material sample to be even larger. Otherwise, the material edge diffractions cause some residual reflections which may fall into the time range prior to the speaker-face reflection. Figure 6.8 (b) and Figure 6.9 (c) indicate temporal locations of the speaker reflection. For the in-situ windowing method, due to the decisive window length given by the Window 1 as the dash-line frame indicated in Figure 6.8 (b), the minimum speaker distance to the material surface should be $D = 2d$, twice of the microphone distance to the sample surface. The minimum speaker distance directly impacts the required sample size.

The measurement setup as shown in Figure 6.8 (a) should be applicable to in-situ measurements, however, the speaker distance and the microphone distance to other obstacles, such as floor, light-fixtures and so on are crucial to the accuracy of the measurement results. The measurement setup has to be implemented within the reflection-free zone, although the speaker-face and sample edge diffractions will inevitably cause reflections within the measured impulse response. But these unwanted reflections can be pushed away beyond the window limit by properly adjusting the speaker distance to the sample under test and the sample size.

For the in-situ subtraction method in addition, the microphone should be placed as close to the plane surface of the material under test as possible, so as not to increase the low frequency limit. But the microphone should not be placed in the surface. In that case, the reflection is unlikely to be measurable. The microphone location at the same time, needs to register reflection from the surface which should not significantly impact the on-set and the amplitude of the incident impulse response (direct sound), often it depends on reflective properties of the material under test. In extreme cases,
when the material under test is highly reflective, the microphone needs to be placed more away from the surface than those of highly absorptive ones. If the microphone distance $d$ is substantially large so as to impact the measurement accuracy, a similar correction strategy as stated in Equation (6.38) should be applied.

### 6.2.4 Oblique Incidence In-Situ Impedance Measurement

Using two microphones with a small separation distance $x_{\Delta}$ acoustic particle velocities can be approximately measured in the sound field under plane wave conditions. Figure 6.13 schematically illustrates a measurement setup according to Allard [1, 2]. Near the material surface, two microphones measure sound pressures $P_1$ and $P_2$ expressed in the frequency domain. Then an averaged sound pressure near the material surface is written as

$$P = \frac{P_1 + P_2}{2}, \tag{6.47}$$

and the particle velocity normal to the material surface, being proportional to a pressure gradient with a negative sign (according to Euler equation (15.1) in Chapter 15), can be approximated as

$$V \approx \frac{P_1 - P_2}{j \omega \rho x_{\Delta}}. \tag{6.48}$$

In this way, both the averaged sound pressure and the normal component of the particle velocity can be considered as experimentally captured near the material surface. The surface impedance of an oblique incidence at angle $\theta$ can then be evaluated in terms of dividing Equation (6.47) by Equation (6.48) as

$$Z_s(\theta) \approx \frac{j \omega \rho x_{\Delta}}{2} \frac{P_1 + P_2}{P_1 - P_2} = \frac{j \omega \rho x_{\Delta}}{2} \frac{1 + H_{12}}{1 - H_{12}}, \tag{6.49}$$

with

$$H_{12} = \frac{P_2}{P_1}. \tag{6.50}$$

Similar to the discussion in Sec. 6.1.1, the microphone separation $x_{\Delta}$ dictates the valid frequency range for the surface impedance measurement of oblique incidence. The upper limit frequency is determined by spatial Nyquist frequency, expressed by

$$f_U = \frac{c}{2 x_{\Delta} \cos \theta}, \tag{6.51}$$

with $c$ being the sound speed. A recent experimental investigation [17] has substantiated this upper limit frequency. In similar fashion, in order to achieve
high accuracy of particle velocity in terms of the gradient approximation towards low frequency, Equation (6.17) should also be useful and can be adopted here for a lower limit frequency

$$f_L > \frac{0.01c}{x_\Delta \cos \theta},$$  \hspace{1cm} (6.52)

In addition, towards the low frequency range, the plane wave condition has to be enforced as well, this may require a large source distance to material surface under test and a sufficient large material sample, similar to the discussion in Sec. 6.2.2.

Active research recently reports on oblique incident impedance measurements including the adaptation of this method using a parametric loudspeaker [24], and application of a particle velocity sensor [17], they essentially rely on the principal method as laid out by Allard [1]. Whereas any attempt using a single microphone near the material surface under test, similar to the measurement setup as discussed in Sec. 6.2.2, shown in Fig. 6.8 is clearly different, since Allard’s method is equivalent to the reflectance measurement, where the incident-angle-dependent reflectance is defined as

$$R(\theta) = \frac{P_r(\theta)}{P_i(\theta)},$$  \hspace{1cm} (6.53)

where $P_i(\theta)$, and $P_r(\theta)$ are incident- and reflected sound pressures in the frequency domain as indicated in Fig. 6.14.
6.3 Further reading

A comprehensive book dealing with the classical methods discussed in this chapter can also be found in [5]. A microphone array method based on spatial Fourier transform for oblique incidence reflection coefficients has been reported [25, 26]. A recent attempt using an open spherical microphone array [11] has also been reported. The investigation is carried out in laboratory environments, based on a model matching approach. Recently, wavenumber spectrum approaches have also been investigated [5, 20], including oblique incidence in situ measurement methods based on the wavenumber spectrum analysis [19].
6.4 Exercises

Problem 6.1.

Figure 6.15 shows a segment of impedance tube. the separation between microphone 1 and 2 is denoted by \( s \), while the distance from microphone 2 to the frontal surface of the material under test is labeled as \( l \). Determine the normal incident surface reflectance.

![Fig. 6.15. Normal incidence plane wave for acoustic reflection coefficient measurement using two microphones [8, 9]. A transfer function \( H_{12} \) is defined between microphone position 1 and 2, separated by distance \( s \), the distance \( l \) from microphone position 2 to the frontal surface of the material.](image1)

Problem 6.2.

Figure 6.16 schematically shows the four-microphone method [23] in a segment of impedance tube. On each sides of a porous material of thickness \( d \), two microphones are deployed with exactly known distance \( l_1, l_2 \) to the material surface and the separation \( s \) between the two microphones. So this measurement setup can be used to measure the characteristic impedance \( Z_c \) and the propagation coefficient \( \gamma \) of the porous material sample.

![Fig. 6.16. Normal incidence plane wave for characteristic impedance and propagation coefficient measurements using four microphones [23].](image2)